SHORT COURSE ON MULTISTAGE STOCHASTIC MIXED INTEGER OPTIMIZATION: THEORY, ALGORITHMS AND APPLICATIONS

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Talk 1. Introduction to multistage stochastic mixed integer optimization with risk neutral and alternative risk averse strategies

We present the basic concepts of mathematical mixed integer optimization under uncertainty in the main parameters. A set of strategies for stochastic optimization based on risk averse measures is also introduced as an alternatives to the optimization of the objective function expected value, i.e., in the so-called risk neutral environment. The risk averse measures to consider whose validity is analyzed in this work are as follows: two-stage mean-risk immunization, two-stage and multistage Value-at-Risk strategy, two-stage and multistage stochastic dominance strategy, and a new measure as a mixture of the multistage VaR & stochastic dominance strategies. The common characteristic of these measures is that they require from the modeler a threshold for the objective function value related to each scenario (the recent ones even allow a set of thresholds) and a failure probability for not reaching it. Uncertainty is represented by a scenario tree and is dealt with by both two-stage and multi-stage stochastic (linear and) mixed integer models with complete recourse. A set of real-life applications and its computational results are also presented.

Talk 2. A parallelized algorithmic framework for solving large-scale multistage stochastic mixed 0-1 problems with nonsymmetric scenario trees

We present a parallelized Branch-and-Fix Coordination algorithm referred to as the PC-BFCMS for solving large-scale multistage mixed 0-1 optimization problems up to optimality under uncertainty. The aim of the parallelization of the algorithm is to reduction the comujting time for problem solving. The parallelization is performed at two levels. The inner level parallelizes the optimization of the MIP submodels attached to the set of scenario clusters that have been created by the modeler-defined break stage to decompose the original problem, where the nonanticipativity constraints are partially relaxed. Several strategies are presented for analyzing the performance of the inner parallel computation based on MPI (Message Passing Interface) threads to solve scenario cluster submodels versus the sequential version of the BFC-MS methodology. The outer level of parallelization defines a set of 0-1 variables, the combinations of whose 0-1 values, referred to as paths (one for each combination), allow independent models to be optimized in parallel, such that each one can itself be internally optimized with the inner parallelization approach. The results of using the outer-inner parallelization are remarkable: the larger the number of paths and MPI threads (in addition to the number of threads that the MIP solver allows to be used), the smaller the elapsed time to obtain the optimal solution. This new approach allows problems to be solved faster, and, can thus solve very large scale problems that could not otherwise be solved by plain use of a state-of the-art MIP solver, or could not be solved by the sequential version of the decomposition algorithm in acceptable elapsed time.

Talk 3. On solving multistage stochastic mixed 0-1 problems with Risk Averse Stochastic Dominance Constraints based strategies

We present an extension to the multistage case two recent risk averse measures (see Gollmer et al., 208, 2011) for two-stage stochastic mixed 0-1 problem solving such that an objective function is maximized in the domain of a feasible region subject to first- and second-order Stochastic Dominance Constraints integer-recourse (SDC) . Given the dimensions of large-scale problems augmented by the new variables and constraints required by these two risk measures, it is unrealistic to solve the problem up to optimality by plain use of MIP solvers. Instead of it, decomposition algorithms of some type should be used. We present an extension of our Branch-and-Fix Coordination algorithm, the so named BFC-MS risk averse SDC, where (besides some important refinements for cutting branches purposes) a special treatment is given to cross scenario cluster constraints that appear in SDC risk measures. A computational experience is presented by comparing the risk neutral approach and the tested risk averse strategies by using variations of a real-life problem in the management of a network infrastructure for natural gas transportation. The performance of the new version of the BFC-MS algorithm versus the plain use of a state-of-the-art MIP solver is also reported.

Talk 4. On multistage mixed 0-1 optimization under a mixture of Exogenous and Endogenous Uncertainty in a risk averse environment

We present a general multistage stochastic mixed 0-1 problem where the uncertainty appears everywhere. Two types of uncertainty are considered, namely, the exogenous uncertainty where the only action to be made by the modeler is to react in order to protecting his decisions and the non-usually treated uncertainty so named endogenous one, where the modeler decisions can change the weights of the potential scenarios to occur. The stochastic model that considers a mixture of both types of uncertainty is converted in a mixed 0-1 Deterministic Equivalent Model. We model optimize the objective function expected value subject to Stochastic Dominance Constraints recourse-integer (SDC) for a set of profiles where both types of uncertainty are treated. An extension of our Branch-and-Fix coordination algorithm, the so named BFC risk averse SDC is presented, where a special treatment is given to the endogenous uncertainty as well as the cross scenario constraints required by the SDC strategies to consider in this work. Some computational experience is presented.