Carsharing and Carpooling optimization – A 5 years research experience

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1. Transport Demand Management

• The objective of TDM measures is to manage more efficiently the existing mobility requirements without building new infra-structure. It is opposite to the “predict and provide” approach which is not sustainable.

• The perspective of TDM includes various strategies that increase travel choices and encourage consumers to use each option for what it does best.

• TDM helps create a more balanced, less automobile dependent transport system (Litman, 1999).

• These strategies include policies, programs, services and products that influence how, why, when and where persons travel to make travel behavior more sustainable.
Carsharing and carpooling: Two similar names for two very different transport options and optimization needs

2. Two different transportation options

• Carpooling and Carsharing lead to two very different optimization perspectives.

• In the case of carpooling our approach began by a simulation approach with a focus on the way groups work every day and how they are formed. We wanted to estimate the probability of finding a match in Lisbon.

• But how could we compute the groups which can be formed out of a set of potential carpooling participants? This is a combinatorial problem and it is where the optimization came up. It does not mean that people act optimally when they make these decisions but by computing this we could find an upper-bound for the matching possibilities.
• In the case of carsharing, the problem is completely different and in fact the subject has produced several interesting optimization problems.
• One of the questions is where to locate the depots in order to capture the best trips for the system.

This is even more important when we consider the one-way carsharing option where there is unbalance on vehicle stocks. How can one optimize the relocation operations which are needed to balance the system? And in this case what is the influence of the depots location? Is there a way of capturing more balanced trips?

3. Carpooling

A conceptual model for carpooling simulation:
• The maximum extra driving time $T_{Extra} i$ user $i$ is willing to accept when picking up colleagues, in addition to the time needed to drive directly from home to the workplace or back;

• The earliest time $e_i$ acceptable for leaving home;

• The latest time $u_i$ acceptable for arriving at work;

• The earliest time $e'i$ acceptable for leaving work;

• The latest time $u'i$ acceptable for getting back home;

• The capacity $Q_i$ of his car, this is the maximum number of persons user $i$ is willing to take in the automobile;

• The maximum distance $DistMAX i$ user $i$ is willing to walk from the server destination to his workplace.

The cost of a pool group $k$:

$$\text{cost}(k) = \begin{cases} \sum_{(i,k) \in E} \min_{\text{path}(i,k)}(l_i) / |k| & \text{if } |k| > 1 \\ 0 & \text{otherwise} \end{cases}$$
The variables:

\( x_{ij}^h k \): Binary variable equal to 1 iff arc \((ij)\) is in the shortest Hamiltonian path of a server \(h\) of a pool \(k\);

\( y_{ik} \): Binary variable equal to 1 iff client \(i\) is in pool \(k\);

\( \xi_i \): Binary variable equal to 1 iff client \(i\) is not pooled with any other client;

\( S_{ih}^k \): Non negative variable denoting the pick-up time of client \(i\) at home by server \(h\). \( S_{ih}^k\) denotes the departure time from home of server \(h\);

\( F_{ih}^k \): Non negative variable denoting the arrival time of each client \(i\) at his workplace when traveling with server \(h\). \( F_{ih}^k\) denotes the arrival time of server \(h\) at his workplace;

\( H_{ih}^k \): Non negative variable denoting the departure time of client \(i\) from his workplace traveling with server \(h\). \( H_{ih}^k\) denotes the departure time of server \(h\) from his workplace;

\( l_{ih}^k \): Non negative variable denoting the arrival time of client \(i\) at home, driven by server \(h\). \( l_{ih}^k\) denotes the arrival time of server \(h\) at home.

The objective function:

\[
R_{CP} = \min \left( \sum_{h \in H} \left( \sum_{i \in I} \frac{\sum_{a \in A} \sum_{k \in K} y_{ih}^k x_{ij}^h}{\sum_{i \in I} y_{ih}^k} \right) + \left( \sum_{h \in H} \left( \sum_{i \in I} \frac{\sum_{a \in A} \sum_{k \in K} y_{ih}^k x_{ij}^h}{\sum_{i \in I} y_{ih}^k} \right) \right) + \sum_{h \in H} p h \right)
\]

Constraints:

\[
\sum_{i \in I} x_{ij}^h = y_{ih}^k \quad i, h \in C, k \in K
\]

Force a client \(i\) to be declared in pool \(k\), if there is a path originated in \(h\) going from \(i\) to \(j\);

\[
\sum_{j \in J} x_{ij}^h = y_{ih}^k \quad j, h \in C, k \in K
\]

Continuity of the paths;

\[
\sum_{a \in A} \sum_{k \in K} x_{ij}^h = 1 \quad i \in C
\]

Force each client to be assigned to a pool or to be penalized;

\[
\sum_{k \in K} x_{ij}^h \leq 1 \quad h \in C, k \in K
\]

Car capacity limitation in each group;

\[
V_i^k = \sum_{a \in A} V_a^k \quad h \in C, k \in K
\]

Disables the possibility of forming groups of only one element;
Divide-and-Conquer algorithm to solve the problem

We have adopted a k-means clustering algorithm for the divide stage, using the following distance function:

\[
d_d = d_1 \times \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (xd_i - xd_j)^2 + (yd_i - yd_j)^2 + 22} \\
\times \sqrt{(e_i - e'_j)^2 + (u_i - u'_j)^2 + (e'_i - e'_j)^2 + (u'_i - u'_j)^2}
\]

- \(x_i, y_i, xd,\) and \(yd\) are the coordinates of the trip origin and destination, respectively;
- \(e_i, u_i, e'_i,\) and \(u'_i\) are the earliest and latest time schedules available for carpooling;
- \(d_1\) and \(d_2\) are the weights for the geographic distance and the schedule distance, respectively. These have to be calibrated for best results.

A GIS application to run the simulations:
Average % of unmatched trips as a function of the number of participants in the carpooling group:

- Geographic variations:
  - Output from the program: red dots are unmatched origins and destinations, while green are matched.
  - We are able to produce a theme map with the probabilities of matching.
• The probability of finding a match for a 40% participation rate in the carpooling club:

![Carpooling Probability Graph](image)

• The probability of finding a match for a 40% participation rate related to the absolute number of trips that it corresponds to:

![Carpooling Trip Graph](image)
• The probability of finding a match for growing percentages of participation rates:

![Graph showing probability of finding a match for growing percentages of participation rates.]

4. Carsharing

• Carsharing typically involves a small to medium fleet of vehicles in several depots around a city, which users can reserve and use with hourly payments. After being used, a car has to be returned to where it was taken from (a two-way rental model).

![Carsharing examples from Lisbon and Porto.]

• These systems are an alternative to private vehicle ownership. Instead of owning one or more vehicles, a household or business accesses a fleet of shared-use autos, benefiting from choosing the one that best fits their needs for a specific objective.
Carsharing has witnessed exponential growth in the past years, and demonstrated notable reductions in vehicle ownership trends at a neighborhood scale (Shaheen, S. A. and A. P. Cohen, 2007).

“My only real complaint about carsharing is that it has one major, built-in, self-limiting inefficiency: because cars must be returned to the spot where you pick them up, you can’t take one-way trips. That means that even if what I really want to do is drive from my house to the store and to a friend’s party, from which I’ll get a ride or return by bus, I’m forced to drive to the party and drive home. (This also automatically makes me the designated driver.)” (“The future of Carsharing”, www.worldchanging.com/)

However some discouraging experiments happened with one-way carsharing: one of the most innovative one-way services in the world was terminated after 6 years. Honda offered one-way trips between any of 21 depots in Singapore with no reservation required and no return time needing to be specified. The service had 2,500 members with access to 100 vehicles in this city (The straits Times, 2008).

As membership grew the company wasn’t able to maintain the service quality which was set initially because everybody expected cars to be available. But in reality, this could not be guaranteed due to one-way trips leaving the system with significant unbalance in vehicle stocks.
2. The optimization perspective

A General Mixed Integer Programming Problem (MIP) formulation for locating carsharing depots.

Sets:

\[ N = \{1, \ldots, W\} \] set of available sites for one-way carsharing depots, where \( W \) is the maximum number of depot locations;

\[ S = \{1, \ldots, K\} \] set of possible sizes for each depot, where \( K \) is the maximum number of depot size categories;

A two-dimensional time-space network:

\[ V = \{t_1, \ldots, t_{T_1}, t_{T_1+1}, \ldots, t_T\} \] representing all the \( N \times T \) nodes with \( T \) as the limit for the optimization period.

Data:

\[ \delta_{ij}^t \] A matrix of travel times, in time steps, dependent on the departure time \( t \) between depot \( i \) and \( j \);

\[ Trips_{ij}^t \] The demand matrices for carsharing vehicles at time instant \( t \) from depot \( i \) to depot \( j \).
Decision variables:

\( y_i^k \): Binary variable for the existence of one depot located in \( i \) of type \( k \) \( \forall i \in N \) and \( k \in S \);

\( D_{ij,t+s} \): Discrete variable for the number of accepted trips between depot \( i \) and \( j \) from time step \( t \) to \( t + s \);

\( R_{ij} \): Number of relocated vehicles between depot \( i \) and \( j \) after the operation period;

\( a_i \): Number of available vehicles at depot \( i \) in instant \( t \);

Auxiliar variable:

\( S_{i,t+1} \): Stock of unused vehicles at each depot \( i \) between instant \( t \) and \( t + 1 \).

The constants are:

\( I \): Income per each time step driven by a client for each trip duration \( I \);

\( C_{m1} \): Cost of maintaining one vehicle per time step driven;

\( C_{m2} \): Cost of maintaining one parking space per day;

\( C_r \): Cost of relocating a vehicle per time step driven;

\( C_v \): Cost of the depreciation of one vehicle per day;

\( p^k \): Number of parking spaces for each depot size \( k \);

For sensitivity Analysis:

\( d \): Minimum share of demand to be attended in percentage of the total daily demand;

\( S_{limit} \): Maximum number of depots to create from the \( N \) available sites.
Objective function:

\[
\text{Max}(P) = \sum_{i \in V} (t_{ij} - C_{mi}) D_{i[i+1]} \delta_{ij} - C_v \sum_{i \in V} R_{ij} \delta_{ij} - C_m2 \sum_{i \in \mathbb{R} \times S} y_i^k p^k - C_v \sum_{i \in \mathbb{R} \times S} a_i
\]

This function maximizes the total daily profit \((P)\) of such carsharing scheme operation, taking into consideration as income the time driven by the customers deducting the maintenance costs, and as costs the vehicle relocation expenses, costs for maintaining each depot according to their size and the costs of the vehicles' depreciation.

Subject to,

\[
a_i = a_{i-1} - \sum_{i \in \mathbb{R} \times V} D_{i[i-1]} \delta_{i+1} + \sum_{i \in \mathbb{R} \times V} D_{i[i+1]} \delta_{i} \quad \forall i \in V
\]

Ensures the conservation of flows at each node at \(i\) and updates the available number of vehicles at each depot from time step \(t - 1\) to time step \(t\);
Computes the stocks available at each station in each cycle from $t$ to $t+1$:

$$S_{i(t+1)} = a_i - \sum_{i \in V} D_{i(t+1)}$$

The number of vehicles available in the morning, instant $1$, must be equal to the number of vehicles left at each station at the end of the operation period, instant $T$, plus the vehicles brought for replacing stocks, minus those which are taken for the same purpose in other depots:

$$a_{i1} = a_{iT} + \sum_{j \in N} R_{ji} - \sum_{j \in N} R_{ij} \forall i \in N$$

Forces the number of vehicles in each depot $i$ at instant $t$ to be less than the depot’s capacity:

$$a_{it} \leq \sum_{k \in S} y_{it}^k \forall i \in N$$

Ensures that the number of accepted trips between depots $i$ and $j$ must be lower than the actual trips:

$$D_{i(t+1),j} \leq \text{Trips}_{ij} \forall i, j \in V$$

Ensures that in a given site there will be a depot of size type $S$, or no depot:

$$\sum_{k \in S} y_{ik}^k \leq 1 \forall i \in N$$
Assures that the satisfied share of demand is above the minimum limit $d$ in percentage;

\[ \sum_{i \in N} y_i^t \leq S_{\text{Limit}} \]

Limits the number of stations to a certain upper limit, if $S_{\text{Limit}} = W$, there is the possibility of having a station in any possible location $N$;

\[ y_i^t = \{0, 1\} \quad \forall \; k \in L \]

$y_i^t$ is binary;

$a_{ij}, D_{ij}$ and $A_{ij}$ are integer $\forall \; i, j \in V$

$R_{ij}$ is integer $\forall \; i, j \in N$

Filtering the Trips Database

We had available a survey from 1994, updated with an extra survey in 2001 of trips in the LMA. These trips were filtered for running the model:

- Trips with origin and destination inside Lisbon;
- Euclidean distance greater than 1km;
- Age between 18 and 55;
- Trip duration greater than 10 min;
- Beginning of the trip after 6 am and before 12 pm;
- Travelers unaccompanied.

This resulted in 1780 filtered surveyed trips, which, having in consideration the sample coefficients, would represent 4% of 39,389 trips inside Lisbon. Thus we are assuming that carsharing would not surpass a 4% mode share of all trips with trip ends in Lisbon.
Parameters

116 possible station location

Other parameters

\( T \): 109 time steps and each time step from \( t \) to \( t + 1 \) is a 10 minutes period;

\( I_t \): It is 2 euros per time step, independently of the trip duration in this experiment;

\( C_{m_1} \): It is the costs of gas and vehicle maintenance, estimated as 0.07 euros per km driven (http://Interfile.de);

\( C_{m_2} \): Daily cost of a parking space in a low price area in Lisbon: 5 euros;

\( C_r \): Based in the average wage in portugal: 2 euros per time step;

\( C_v \): Computed through a software for estimating the depreciation of vehicles. Estimated to be 17.35 euros per day (http://Interfile.de)

\( p^k \): \{1,3,5,15,35\}

\( d \) and \( S_{\text{limit}} \) to be varied for sensitivity analysis;
Travel time in Lisbon

VISUM modeling of the LMA main network assigning all automobile traffic and producing three typical travel times: morning and afternoon peak and between peaks. Vector $\delta_i^j$ in time steps of 10 min

5. Experimental design

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\[ S_{\text{lim}} = 10 \]

- 10 stations
- \( P=31.48 \) Euros/day
- 4 vehicles
- 70 trips (4%)

\[ S_{\text{lim}} = 20 \]

- 20 stations
- \( P=94.98 \) Euros/day
- 9 vehicles
- 155 trips (9%)
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\( S_{\text{limit}} = 30 \)

- 30 stations
- \( P=176.86 \) Euros/day
- 14 vehicles
- 257 trips (14%)

\( S_{\text{limit}} = 40 \)

- 40 stations
- \( P=268.77 \) Euros/day
- 20 vehicles
- 374 trips (21%)
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\[ S_{\text{limit}} = 50 \]

- 50 stations
- \( P = 332.11 \) Euros/day
- 27 vehicles
- 476 trips (27%)

\[ S_{\text{limit}} = 60 \]

- 60 stations
- \( P = 367.83 \) Euros/day
- 31 vehicles
- 534 trips (30%)
Optimum Solution

- 66 stations
- \( P = 376.91 \) Euros/day
- 33 vehicles
- 561 trips (32%)

D=40%

- 74 stations
- \( P = 330.06 \) Euros/day
- 44 vehicles
- 711 trips (40%)


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**D=60%**

- 91 stations
- $P = -224.14$ Euros/day
- 89 vehicles
- 1067 trips (60%)

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**D=80%**

- 104 stations
- $P = -2398.97$ Euros/day
- 201 vehicles
- 1423 trips (80%)
d=100%

- 108 stations
- $P=9758$ Euros/day
- 421 vehicles
- 1780 trips (100%)

Different objectives would produce different optimum solutions
Where are the best locations for placing a carsharing depot?

Depots in different places of the city capture different demand patterns of trips, here we see the example of a depot which is placed in the outskirts and another in the city centre.

Vehicle Stocks in each depot during a working day

- 10 depots

Each color is the stock of vehicles in a different depot.

We may observe that there are cycles in which stocks increase and decrease.
• All demand must be attended (108 stations; 100% demand attended)

Cycles are very smooth and most of the time there is a very high number of vehicles in stock in each depot, which is very inefficient.

Generalizing the model

• Several model specification variations may be introduced in order to try to translate other possibilities of the carsharing system functioning.

• In the present formulation we started from the principle that persons are only willing to use one depot for the origin and another for the destination, this is always the one which is closer to the point coordinates (using the pedestrian network).

• What if people are willing to be flexible and pick-up a vehicle at another depot which is not so close but it is still close enough for him to be willing to use this alternative?

• We call these the flexible travelers, and this may have an impact on profit.
This is the current approach – No flexibility and trips concentrated at the nearest depot

- People are flexible and may pick-up or drop-off a vehicle in one of the three closest depots to the origin or destination point.
Workshop – Transportation Optimization
Coimbra, Portugal/ November 26, 2010
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- People are flexible and may pick-up or drop-off a vehicle in one of the three closest depots to the origin or destination point. Besides, they have perfect information of vehicle stocks in each of the three possibilities and only go to the one that may serve them (relies in a centralized information center).

*Results*

- Lucro
- Média por veículo (km/veículo)
- Percentagem da média de tempo que os veículos estão em utilização
- Procura satisfeita

- inflexível
- flexível com informação completa
- flexível sem informação
The next step: simulation

MIP has the following major problems in translating reality:

- In the real world trips are not deterministic, they are intrinsically stochastic.
- We never know what’s going to happen in the rest of the day, thus optimizing for a full business day may produce unrealistic results.
- It is difficult to test a full dynamic price policy, which may play an important role in balancing the system.
- Simulation allows to easily model two different objectives: the Client’s and Business objectives: the client wants to travel fast and cheap. The company wants to maximize its profit.
- Integrating the results of the optimization and simulation one is able to test the optimality of the solution with trip uncertainty.

Questions and/or Suggestions?

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