

# Some rankings for the Athens Olympic Games using DEA models with a constant input

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## Abstract

There is no official method to establish a final ranking for the Olympic games. The usual ranking is based on the Lexicographic Multicriteria Method, the main drawback of which is to overvalue gold medals. Furthermore it does not take in account that the various sports may be of different importance. This work proposes a ranking model to eliminate those drawbacks. First we use a modified cross evaluation DEA (Data Envelopment Analysis) model with weighted restrictions for each sport. The outputs are the number of gold, silver and bronze medals and the input is a unitary constant for all countries. After obtaining a rank for each and every sport we build a general ranking using a weighted sum. The weights are calculated taking in account the number of countries that participated in each sport. We use our model with the results of the Athens Olympic Games.

**Keywords:** DEA, Olympic, Ranking, weight restrictions, unitary input

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## 1 Introduction

The first recorded Olympic Games occurred in Olympia, Greece in 776 ac (Fischer, 2003). The modern Games, were born from an initiative of Baron de Coubertin in 1892 and occurred in 1896 in Athens. The Games tried to maintain the initial spirit of individual competition. As noted by Lins et al (2003) the purpose clearly failed and the Games have become a national competition.

Despite their national character, the Olympic Committee has never issued an official ranking to pick an overall Olympic winner country. However the IOC presents the medals data in a table that suggests a ranking. As a matter of fact, the mass media do use this table as a ranking. This quasi-official ranking is based on the Lexicographic Multicriteria method, as explained in Lins et al (2003). This ranking does not deal properly with the

possible existence of countries that have won a large number of silver and bronze medals but no gold medal.

In the literature there are some studies for alternative rankings for the Olympic Games. In almost all of them the rankings were obtained by comparing the number of medals earned with explicative variables. In all of them there is no difference in the value of medals earned in different competitions.

We have two main goals in this study. The first one is to provide a rank (different from the quasi-official ranking) that takes into account only the number of medals won and not the explicative variables. This ranking must not over evaluate the gold medal. We must mention that there are already at least two rankings with this characteristic. We believe that our ranking has some theoretical advances in the DEA field.

The second and most important goal is to take into account that medals won in different competitions do not have the same value. As a matter of fact, the existing rankings do not take into account that in some disciplines there are more events than in others, and so there are more possibilities of winning a medal. For instance, in gymnastics there are a lot of gold medals to be earned and in football there are only two possibilities for a country to win a gold medal (one for men, the other for women). As far as we know this the first study on that matter at least concerning Olympic rankings.

To take into account the difference in winning values for different sports, we aggregate competitions into clusters, as done by the IOC ([www.olympic.org](http://www.olympic.org)): each discipline is a cluster.

Our method is illustrated using data from the 2004 Athens Olympic Games.

In the next section we review the literature about Olympic rankings and the literature of Data Envelopment Analysis rankings in sports. In section 3, we establish the fundamentals of our ranking, the rankings for each sport and the two general rankings. At last, section 4 presents some conclusions concerning the results we have obtained.

## **2 A Review on the Analysis of the Olympic Games results**

The Lexicographic Method is not the sole method used to rank countries in the Olympic Games. Some newspapers produce a ranking determining the total number of medals earned by each country. They simply add up bronze, silver and gold medals. The obvious drawback of this method is to under-evaluate gold medals.

An alternative approach is to make an arbitrary evaluation of each medal, for instance, 1 point for bronze, 2 for silver and 3 for gold. This is a much unsophisticated approach, as it assumes all medals to be equally desired, albeit in proportion to their value.

The previous approaches follow contradictory assumptions. It is important to study alternative ways to rank competitors in the Olympic Games. Morton (2002) used statistical methods to determine “who wins the Olympics”. Other statistical approach considering socio economical variables and the number of medal earned has been performed by Bernard and Busse (2004).

There are already some approaches using DEA to establish Olympic rankings. The very first one was proposed by Lozano et al (2002). They used population and GNP as inputs and the medals as outputs. In a similar approach, Lins et al (2003) built a new model taking in account one more constraint: the total amount of medals is a constant. This resulted in the development of a new model, the so-called Zero Sum Gains DEA model (ZSG-DEA). Churilov and Flitman (2006) used DEA to establish a ranking, the inputs of which were some social economics variables. Instead of using as outputs the number of gold, silver and bronze medal, they used four linear combinations of these figures. This approach eliminates the problem of nil valued weights. For each country, they determined which output has the greatest weight, in order to divide the countries into clusters. They also made a classical cluster analysis using socio economical

variables, and compared the two classifications. The authors emphasized the importance of Olympic rankings and asked for new studies on this subject.

All the works mentioned hereabove take into account the results in the Olympics and the socio economical conditions of each country.

Balmer et al (2001); Balmer et al (2003), Soares de Mello et al (2004) and Soares de Mello et al (2008) use only the results themselves. The former study the existence of "home advantage" and the latter want to establish a ranking for both Summer and Winter Games. Another work comparing summer and winter Games is the one of Johnson and Ali (2004). The uncertainty in Olympic Games was studied by Baimbridge (1998).

In all the above mentioned papers there is no difference in the value of a medal in different sports, i.e., the one hundred meters gold medal has the same value of a baseball gold medal. In this paper we will propose a method that takes into account that the medals are obtained in different sports.

We shall also mention that there are DEA based rankings for some other sports. Among them we can mention Espitia-Escuer and Garci-Cebrian (2006), Barros and Leach (2006), Haas (2003), Soares de Mello and Gomes Junior (2006) and Calôba and Lins (2006).

### 3 Building the new ranking

Our method is performed in two steps. The first one is to make a ranking for each sport independently. This is done to avoid the possibility of a country that has a good performance in a sport that has a great number of different competitions (athletics, gymnastics, swimming and so on) to be placed in a higher position than a country with a similar performance in a sport with a few number of different competitions (baseball, football, volleyball, and so on). In the second step, we must aggregate the different rankings obtained in step one. This is achieved using a weighted sum of the partial performances. We mention different forms to determine these weights and we carry out the calculation for two of them.

For the first step, we will use a DEA model for each sport. The DMUs (Decision Making Units that are the units under evaluation in DEA) are all the countries that won a medal in this sport. The three outputs are the number of gold, silver and bronze medal each country earned. We do not use inputs. According to Lovell and Pastor (1999) this leads to mathematical inconsistencies and so we adopt a unitary input for each DMU.

Owing to the existence of a single constant input, we use the Constant Returns to Scale DEA model (DEA CCR) Charnes et al (1978). In the particular case of a constant input the CCR model becomes (1) in which  $h_0$  is the DMU 0 efficiency;  $y_{jk}$  is the j-th output ( $j=1, \dots, s$ ) of the k-th DMU ( $k=1, \dots, n$ );  $x_{ik}$  is the i-th input ( $i=1, \dots, r$ ) of the k-th DMU;  $\mu_j$  and  $\nu_i$  are the output and the input weights, respectively. As we have a unitary input it does not appear in the formulation.

$$\begin{aligned}
 & \text{Maximise } h_0 = \sum_{j=1}^s \mu_j y_{j0} \\
 & \text{subject to} \\
 & \sum_{j=1}^s \mu_j y_{jk} \leq 1, \quad k = 1, \dots, n \\
 & \mu_j \geq 0, \quad j = 1, \dots, s,
 \end{aligned} \tag{1}$$

Although this is an input oriented DEA model, this model allows other interpretation owing to the absence of the equality constraint. If this model was to be used with an input oriented interpretation, this model will become meaningless owing to the presence of a unitary input. The dual for this model is presented in (2)

$$\begin{aligned}
 & \text{Minimize } \sum_{k=1}^n \lambda_k \\
 & \text{subject to} \\
 & \sum_{k=1}^n y_{jk} \lambda_k \geq y_{j0}, \quad j=1, \dots, s \\
 & \lambda_k \geq 0, \quad \forall k
 \end{aligned} \tag{2}$$

In this model there is no input reduction. The minimization of the share sum interpretation makes the model a meaningful one even in the presence of a constant input. This model has already been derived by Caporaletti et al (1999). The authors interpreted this model as a multi-attribute one, in the spirit of DEA only with outputs. This is the same as considering a unitary and constant input. Foroughia and Tamiz (2005) use an analogous model but they missed the theoretical considerations. A model with the same objective function and different constraints is used by Kao and Hung (2007).

For the Olympic ranking, model (1) is transformed into model (3), where  $g$ ,  $s$  and  $b$  refer to the gold, silver and bronze medals in the Athens 2004 Olympic Games.

$$\begin{aligned}
 & \text{Maximise } h_0 = \mu_g y_{g0} + \mu_s y_{s0} + \mu_b y_{b0} \\
 & \text{subject to} \\
 & \mu_g y_{gk} + \mu_s y_{sk} + \mu_b y_{bk} \leq 1, \quad k = 1, \dots, n \\
 & \mu_j \geq 0, \quad j = g, s, b
 \end{aligned} \tag{3}$$

Obviously the medals are not equally important. To take that fact into account we will use weight restrictions in our DEA model. For sure, a gold medal is more important than a silver one and this one is more important than a bronze one. However, the difference in their relative importance is not the same. In opposition to Baron de Coubertin ideals, victory is the main goal of the competitors. So the difference in importance between gold and silver medals should be greater than the difference between silver and bronze medals. Having these assumptions in mind, the unitary input DEA model based on Soares de Mello and Gomes Junior (2006) model is shown in (4).

$$\begin{aligned}
 & \text{Maximise } h_0 = \mu_g y_{g0} + \mu_s y_{s0} + \mu_b y_{b0} \\
 & \text{subject to} \\
 & \mu_g y_{gk} + \mu_s y_{sk} + \mu_b y_{bk} \leq 1, \quad k = 1, \dots, n \\
 & \mu_g - \mu_s \geq 0.001 \\
 & \mu_s - \mu_b \geq 0.001 \\
 & \mu_g - 2\mu_s + \mu_b \geq 0.001 \\
 & \mu_j \geq 0, \quad j = g, s, b
 \end{aligned} \tag{4}$$

This is a weight restrictions model with non-homogeneous restrictions; such models were studied by Podinovsky (2004)

The non Archimedean constant 0.001 is required to avoid a critical distortion, i.e. in special conditions the three medals may be equally valued. Such a situation leads to non-suitable rankings. For instance, in table 1, medals for Baseball are shown.

Table 1. Medals for Baseball

| Country   | Gold | Silver | Bronze |
|-----------|------|--------|--------|
| Cuba      | 1    | 0      | 0      |
| Australia | 0    | 1      | 0      |
| Japan     | 0    | 0      | 1      |

Source: International Olympic Committee

As shown in that table each one of the three countries earned only one medal. If we applied the model with homogenous weights restrictions, the three countries will be equally efficient. This is not a desirable result. Common sense will attribute the first position for Cuba, the second one for Australia and the third place to Japan.

Even with the constraints imposed in this model, there is a high degree of freedom for the weights. To avoid this freedom, we use a secondary model inspired on the Sexton et al (1986) Cross Evaluation model. As we have only one input, the cross evaluation model becomes a fixed weight model Anderson et al (2002). So, we use average weights, which are easier to calculate than using the aggressive and benevolent model of Doyle and Green (1994). Average weights are used in a model similar to cross evaluation used by Lins et al (2003). The use of both weight restrictions and cross evaluation combines two approaches for improving discrimination in DEA: the first one with decision maker value judgements and the second one is a fully objective one (Angulo-Meza and Lins, 2002)

On the other hand, it is common knowledge in DEA that for some DMUs the weights are not uniquely determined (see for instance, Rosen et al, (1998); Soares de Mello et al, (2002); Cooper et al, (2007)). We use an auxiliary linear programming model to determine a unique set of weights for each DMU. The aim of this model is to maximize the difference of the weights for gold and silver medals assuming that the efficiency previously determined in model (4) remains the same. This model is shown in (5).

$$\begin{aligned}
 & \text{Maximise } \mu_g - \mu_s \\
 & \text{subject to} \\
 & \mu_g y_{g0} + \mu_s y_{s0} + \mu_b y_{b0} = h_o, \\
 & \mu_g y_{gk} + \mu_s y_{sk} + \mu_b y_{bk} \leq 1, \\
 & \mu_g - \mu_s \geq 0.001 \\
 & \mu_g - \mu_s \geq 0.001 \\
 & \mu_g - 2\mu_s + \mu_b \geq 0.001 \\
 & \mu_j \geq 0, \quad j = g, s, b
 \end{aligned} \tag{5}$$

The objective function guarantees the maximization of the difference between the importance of the gold and silver medals. The invariance of the efficiency value is assured by the first constraint. The remaining constraints are the same as used in model (4).

In each sport, to obtain the gold medal weight we calculate the average of all the weights attributed to the gold medal by the complete set of DMUs. In a similar way we obtain the weights for silver and bronze medals. The performance of a particular DMU in a given sport is calculated using equation (6).

$$P_o = \overline{\mu}_g Y_g + \overline{\mu}_s Y_s + \overline{\mu}_b Y_b \tag{6}$$

### 3.1 Some results for the first step

Using models (4) and (5) and equation (6), we obtain the results for every sport. In table 2 we show results for Sailing.

Table 2. Sailing results

| DMU                    | $y_g$ | $y_s$ | $y_b$ | $\mu_g$  | $\mu_s$  | $\mu_b$  | $P_0$    |
|------------------------|-------|-------|-------|----------|----------|----------|----------|
| United Kingdom         | 2     | 1     | 2     | 0,49995  | 0,0001   | 0        | 1        |
| Brazil                 | 2     |       |       | 0,49995  | 0,0001   | 0        | 0,710103 |
| Spain                  | 1     | 2     |       | 0,40002  | 0,19996  | 0        | 0,655042 |
| Austria                | 1     | 1     |       | 0,40002  | 0,19996  | 0        | 0,505047 |
| Greece                 | 1     | 1     |       | 0,40002  | 0,19996  | 0        | 0,505047 |
| USA                    | 1     | 1     |       | 0,40002  | 0,19996  | 0        | 0,505047 |
| France                 | 1     |       | 1     | 0,49995  | 0,0001   | 0        | 0,425003 |
| Israel                 | 1     |       |       | 0,49995  | 0,0001   | 0        | 0,355052 |
| Norway                 | 1     |       |       | 0,49995  | 0,0001   | 0        | 0,355052 |
| Ukraine                |       | 2     |       | 0,40002  | 0,19996  | 0        | 0,29999  |
| Canada                 |       | 1     |       | 0,40002  | 0,19996  | 0        | 0,149995 |
| China                  |       | 1     |       | 0,40002  | 0,19996  | 0        | 0,149995 |
| Czech Republic         |       | 1     |       | 0,40002  | 0,19996  | 0        | 0,149995 |
| Denmark                |       |       | 2     | 0,20016  | 0,19996  | 0,19986  | 0,139902 |
| Argentina              |       |       | 1     | 0,20016  | 0,19996  | 0,19986  | 0,069951 |
| Italy                  |       |       | 1     | 0,20016  | 0,19996  | 0,19986  | 0,069951 |
| Japan                  |       |       | 1     | 0,20016  | 0,19996  | 0,19986  | 0,069951 |
| Poland                 |       |       | 1     | 0,20016  | 0,19996  | 0,19986  | 0,069951 |
| Slovenia               |       |       | 1     | 0,20016  | 0,19996  | 0,19986  | 0,069951 |
| Sweden                 |       |       | 1     | 0,20016  | 0,19996  | 0,19986  | 0,069951 |
| <i>Average weights</i> |       |       |       | 0,355052 | 0,149995 | 0,069951 |          |

Table 3. The results for Archery

| DMU                    | $y_g$ | $y_s$ | $y_b$ | $\mu_g$  | $\mu_s$  | $\mu_b$  | $P_0$    |
|------------------------|-------|-------|-------|----------|----------|----------|----------|
| Korea                  | 3     | 1     |       | 0,3333   | 0,0001   | 0        | 1        |
| Italy                  | 1     |       |       | 0,3333   | 0,0001   | 0        | 0,270863 |
| Chinese Taipei         |       | 1     | 1     | 0,25005  | 0,24985  | 0,24975  | 0,374725 |
| China                  |       | 1     |       | 0,25005  | 0,24985  | 0,24975  | 0,187413 |
| Japan                  |       | 1     |       | 0,25005  | 0,24985  | 0,24975  | 0,187413 |
| Australia              |       |       | 1     | 0,25005  | 0,24985  | 0,24975  | 0,187313 |
| United Kingdom         |       |       | 1     | 0,25005  | 0,24985  | 0,24975  | 0,187313 |
| Ukraine                |       |       | 1     | 0,25005  | 0,24985  | 0,24975  | 0,187313 |
| <i>Average weights</i> |       |       |       | 0,270863 | 0,187413 | 0,187313 |          |

As shown in table 2, in the case of sailing, the ranking obtained by our model and the lexicographic method is the same. A case where the two methods lead to different rankings is Archery, whose results are shown in table 3.

Table 4 shows the average weights for all disciplines as well as the differences between average weights.

Table 4. Medals Average Weights for every discipline.

| Discipline                | $\bar{\mu}_g$ | $\bar{\mu}_s$ | $\bar{\mu}_b$ | $\bar{\mu}_g - \bar{\mu}_s$ | $\bar{\mu}_s - \bar{\mu}_b$ |
|---------------------------|---------------|---------------|---------------|-----------------------------|-----------------------------|
| Archery                   | 0.270863      | 0.187413      | 0.187313      | 0.083450                    | 0.000100                    |
| Athletics                 | 0.078268      | 0.022021      | 0.021921      | 0.056248                    | 0.000100                    |
| Badminton                 | 0.222300      | 0.166600      | 0.166500      | 0.055700                    | 0.000100                    |
| Baseball                  | 1.000000      | 0.666567      | 0.666467      | 0.333433                    | 0.000100                    |
| Basketball                | 0.900030      | 0.300010      | 0.099970      | 0.600020                    | 0.200040                    |
| Beach volleyball          | 0.750000      | 0.250000      | 0.249900      | 0.500000                    | 0.000100                    |
| Boxing                    | 0.128831      | 0.118649      | 0.118549      | 0.010182                    | 0.000100                    |
| Canoe/kayak flatwater     | 0.161822      | 0.117571      | 0.117471      | 0.044251                    | 0.000100                    |
| Canoe/kayak slalom racing | 0.388917      | 0.222167      | 0.222067      | 0.166750                    | 0.000100                    |
| Cycling mountain bike     | 1.000000      | 0.666567      | 0.664667      | 0.333433                    | 0.001900                    |
| Cycling road              | 0.703722      | 0.185211      | 0.111067      | 0.518511                    | 0.074144                    |
| Cycling track             | 0.132667      | 0.086989      | 0.081343      | 0.045678                    | 0.005646                    |
| Diving                    | 0.127034      | 0.079306      | 0.079206      | 0.047728                    | 0.000100                    |
| Equestrian                | 0.437398      | 0.125204      | 0.101746      | 0.312195                    | 0.023458                    |
| Fencing                   | 0.227012      | 0.082541      | 0.071343      | 0.144471                    | 0.011198                    |
| Football                  | 1.000000      | 0.666567      | 0.666467      | 0.333433                    | 0.000100                    |
| Gymnastics Artistic       | 0.180873      | 0.049377      | 0.042793      | 0.131496                    | 0.006584                    |
| Gymnastics Rythmic        | 0.375038      | 0.249925      | 0.249825      | 0.125113                    | 0.000100                    |
| Trampoline                | 0.900030      | 0.300010      | 0.099970      | 0.600020                    | 0.200040                    |
| Handball                  | 1.000000      | 0.666567      | 0.666467      | 0.333433                    | 0.000100                    |
| Hockey                    | 0.875038      | 0.250025      | 0.124963      | 0.625013                    | 0.125063                    |
| Judo                      | 0.103157      | 0.087373      | 0.087273      | 0.015784                    | 0.000100                    |
| Modern Pentathlon         | 1.000000      | 0.666567      | 0.666467      | 0.333433                    | 0.000100                    |
| Rowing                    | 0.282669      | 0.173878      | 0.166541      | 0.108791                    | 0.007338                    |
| Sailing                   | 0.355052      | 0.149995      | 0.069951      | 0.205057                    | 0.080044                    |
| Shooting                  | 0.181849      | 0.063653      | 0.036339      | 0.118196                    | 0.027314                    |
| Softball                  | 1.000000      | 0.666567      | 0.666467      | 0.333433                    | 0.000100                    |
| Swimming                  | 0.047813      | 0.028005      | 0.024888      | 0.019808                    | 0.003117                    |
| Sync swimming             | 0.500000      | 0.333233      | 0.333133      | 0.166767                    | 0.000100                    |
| Table tennis              | 0.200100      | 0.133300      | 0.133200      | 0.066800                    | 0.000100                    |
| Taekwondo                 | 0.365427      | 0.173088      | 0.134565      | 0.192338                    | 0.038523                    |
| Tennis                    | 0.383403      | 0.233293      | 0.233193      | 0.150110                    | 0.000100                    |
| Triathlon                 | 0.700020      | 0.299980      | 0.299880      | 0.400040                    | 0.000100                    |
| Volleyball                | 1.000000      | 0.333387      | 0.266653      | 0.666613                    | 0.066733                    |
| Water polo                | 1.000000      | 0.666567      | 0.666467      | 0.333433                    | 0.000100                    |
| Weightlifting             | 0.140048      | 0.099920      | 0.099820      | 0.040128                    | 0.000100                    |
| Wrestling                 | 0.133407      | 0.071191      | 0.063528      | 0.062216                    | 0.007663                    |

The difference between the average weights for gold and silver is larger than the difference between silver and bronze, i.e.,  $\bar{\mu}_g - \bar{\mu}_s > \bar{\mu}_s - \bar{\mu}_b$ . Although this is an obvious consequence of this constraint, it can be seen that for almost all the disciplines,  $\bar{\mu}_g - \bar{\mu}_s \gg \bar{\mu}_s - \bar{\mu}_b$ . This means that for the majority of countries the gold medal is much more important than the other medals.

On other hand, for a large number of disciplines,  $\bar{\mu}_s - \bar{\mu}_b$  is very small. For 21 disciplines, this difference is 0.0001. This is the value chosen for the non-Archimedean

constant  $\varepsilon$ . So we conclude that this constant is very important to distinguish the weight values for silver and bronze medals. As a matter of fact, we may conclude that for a large number of countries winning a silver medal or one of bronze has almost the same value.

### 3.2 Aggregation of the partial results

As mentioned in the introduction of this paper, the final ranking is obtained using a weighted sum of the performance of each country in each sport as shown in equation (7).

$$I_o = \sum_{i=1}^{37} P_{oi} n_i \quad (7)$$

In which  $P_{oi}$  is the performance of the country  $O$  in sport  $i$  and  $n_i$  are the weights for sport  $i$ ,  $i = 1, \dots, 37$ .

Different methods can be used to estimate the weights for each sport. In a first model, we can suppose that all sports are equally important and so the weights are all the same.

In a second model, we can measure the importance of each sport for their potential of attracting spectators, mainly when TV broadcastings are concerned. A direct approach would need the figures for TV audiences. This is a rather difficult task, so we may use as a proxy the number of countries participating in each sport. Despite the several drawbacks of this approach, we are justified to believe that the greater the number of countries participating in a sport, the greater the number of potential spectators.

Another method would weigh each sport according to its competitiveness Mitchell and Stewart (2007).

In this paper, we will use both the first and the second model taking into account the number of participating countries rather than TV audiences.

The number of participant countries in each sport is shown in table 5.

After using equation (7) to obtain the indexes, these are normalized using equation (8).

$$Index_o = \frac{I_o}{I_{max}} * 100 \quad (8)$$

The results for both models are shown in table 6 and 7.



Table 5. Participant countries for each sport

| Sport                     | n <sub>i</sub> | Sport             | n <sub>i</sub> |
|---------------------------|----------------|-------------------|----------------|
| Archery                   | 43             | Handball          | 16             |
| Athletics                 | 196            | Hockey            | 14             |
| Badminton                 | 32             | Judo              | 94             |
| Baseball                  | 32             | Modern Pentathlon | 26             |
| Basketball                | 18             | Rowing            | 55             |
| Beach volleyball          | 24             | Sailing           | 61             |
| Boxing                    | 72             | Shooting          | 106            |
| Canoe/kayak flat-water    | 45             | Softball          | 8              |
| Canoe/kayak slalom racing | 22             | Swimming          | 154            |
| Cycling mountain bike     | 34             | Sync swimming     | 24             |
| Cycling road              | 49             | Table tennis      | 50             |
| Cycling track             | 39             | Tae-kwon-do       | 60             |
| Diving                    | 30             | Tennis            | 53             |
| Equestrian                | 68             | Triathlon         | 33             |
| Fencing                   | 42             | Volleyball        | 19             |
| Football                  | 22             | Water polo        | 13             |
| Gymnastics Artistic       | 42             | Weightlifting     | 79             |
| Gymnastics Rhythmic       | 21             | Wrestling         | 99             |
| Trampoline                | 19             |                   |                |

Table 6. Final Ranking using identical weights for all sports

| Ranking | Country        | Index  | Ranking | Country         | Index    |
|---------|----------------|--------|---------|-----------------|----------|
| 1       | USA            | 100,00 | 39      | Belgium         | 4,321921 |
| 2       | Russia         | 90,95  | 40      | Uzbekistan      | 4,272584 |
| 3       | Germany        | 71,21  | 41      | North Korea     | 3,533214 |
| 4       | Australia      | 54,34  | 42      | Azerbaijan      | 3,292518 |
| 5       | Italia         | 46,80  | 43      | Israel          | 3,286145 |
| 6       | France         | 42,13  | 44      | Ireland         | 3,24955  |
| 7       | Korea          | 37,37  | 45      | Mexico          | 3,095507 |
| 8       | United Kingdom | 37,01  | 46      | Georgia         | 2,984847 |
| 9       | Japan          | 34,29  | 47      | Slovenia        | 2,622701 |
| 10      | Ukraine        | 28,83  | 48      | South Africa    | 2,312637 |
| 11      | Netherlands    | 27,68  | 49      | Estonia         | 2,103016 |
| 12      | Hungary        | 27,64  | 50      | Cuba            | 1,99198  |
| 13      | Brazil         | 27,48  | 51      | China           | 1,982228 |
| 14      | Greece         | 18,43  | 52      | Ethiopia        | 1,97946  |
| 15      | Argentina      | 17,48  | 53      | Venezuela       | 1,741313 |
| 16      | Canada         | 16,57  | 54      | Spain           | 1,733198 |
| 17      | Romania        | 15,64  | 55      | Portugal        | 1,702438 |
| 18      | Norway         | 14,82  | 56      | Jamaica         | 1,652263 |
| 19      | New Zealand    | 11,39  | 57      | Kenya           | 1,561582 |
| 20      | Czech Republic | 10,49  | 58      | U Arab Emirates | 1,351006 |
| 21      | Bulgaria       | 10,42  | 59      | Morocco         | 1,326551 |
| 22      | Belarus        | 10,41  | 60      | Finland         | 1,001795 |
| 23      | Austria        | 10,29  | 61      | Egypt           | 0,991114 |
| 24      | China Taipei   | 9,50   | 62      | Hong Kong       | 0,990322 |

|    |             |         |    |                     |          |
|----|-------------|---------|----|---------------------|----------|
| 25 | Slovakia    | 9,22    | 63 | Denmark             | 0,989579 |
| 26 | Turkey      | 7,67    | 64 | Syria               | 0,880729 |
| 27 | Chile       | 7,43    | 65 | Zimbabwe            | 0,748165 |
| 28 | Thailand    | 7,28    | 66 | Bahrain             | 0,744332 |
| 29 | Switzerland | 7,24    | 67 | Colombia            | 0,74159  |
| 30 | Iran        | 6,28    | 68 | Croatia             | 0,74159  |
| 31 | Poland      | 6,28    | 69 | Mongolia            | 0,648371 |
| 32 | Latvia      | 6,22    | 70 | Cameroon            | 0,581476 |
| 33 | Lithuania   | 5,70    | 71 | Dominican Republic  | 0,581476 |
| 34 | SCG         | 5,43    | 72 | India               | 0,472897 |
| 35 | Kazakhstan  | 5,15    | 73 | Nigeria             | 0,325711 |
| 36 | Paraguay    | 4,95    | 74 | Trinidad and Tobago | 0,184898 |
| 37 | Indonesia   | 4,87    | 75 | Eritrea             | 0,162856 |
| 38 | Sweden      | 4,75113 |    |                     |          |

Table 7. Final Ranking using different weights for all sports

| Ranking | Country        | Index  | Ranking | Country            | Index |
|---------|----------------|--------|---------|--------------------|-------|
| 1       | USA            | 100,00 | 39      | Egypt              | 5,77  |
| 2       | Russia         | 88,08  | 40      | Jamaica            | 5,37  |
| 3       | China          | 65,79  | 41      | Kenya              | 5,07  |
| 4       | Germany        | 52,19  | 42      | Slovakia           | 4,85  |
| 5       | Australia      | 41,67  | 43      | Azerbaijan         | 4,68  |
| 6       | Japan          | 35,75  | 44      | Lithuania          | 4,55  |
| 7       | France         | 35,12  | 45      | Georgia            | 4,44  |
| 8       | United Kingdom | 34,67  | 46      | North Korea        | 4,33  |
| 9       | Italy          | 34,12  | 47      | Morocco            | 4,31  |
| 10      | South Korea    | 32,08  | 48      | Switzerland        | 4,22  |
| 11      | Cuba           | 30,53  | 49      | Belgium            | 4,18  |
| 12      | Ukraine        | 25,13  | 50      | South Africa       | 4,10  |
| 13      | Netherlands    | 24,84  | 51      | Latvia             | 3,89  |
| 14      | Greece         | 18,26  | 52      | Israel             | 3,68  |
| 15      | Spain          | 18,08  | 53      | Ireland            | 3,66  |
| 16      | Hungary        | 17,98  | 54      | Slovenia           | 3,24  |
| 17      | Brazil         | 16,03  | 55      | Mexico             | 3,22  |
| 18      | Romania        | 14,93  | 56      | Indonesia          | 3,16  |
| 19      | Belarus        | 13,76  | 57      | Estonia            | 2,72  |
| 20      | Norway         | 12,20  | 58      | Bahamas            | 2,42  |
| 21      | Bulgaria       | 11,39  | 59      | U Arab Emirates    | 2,37  |
| 22      | Canada         | 11,02  | 60      | Portugal           | 2,18  |
| 23      | Turkey         | 10,33  | 61      | Venezuela          | 1,97  |
| 24      | Austria        | 9,18   | 62      | Zimbabwe           | 1,91  |
| 25      | Thailand       | 8,91   | 63      | Serbia/Montenegro  | 1,90  |
| 26      | Poland         | 8,82   | 64      | Cameroon           | 1,89  |
| 27      | Sweden         | 8,81   | 65      | Dominican Republic | 1,89  |
| 28      | Czech Republic | 8,76   | 66      | Paraguay           | 1,81  |
| 29      | Chinese Taipei | 8,66   | 67      | Finland            | 1,70  |
| 30      | Iran           | 7,57   | 68      | Nigeria            | 1,06  |
| 31      | Denmark        | 7,47   | 69      | Syria              | 1,05  |
| 32      | Argentina      | 7,44   | 70      | Mongolia           | 1,01  |

|    |             |      |    |                     |      |
|----|-------------|------|----|---------------------|------|
| 33 | New Zealand | 7,27 | 71 | Colombia            | 0,97 |
| 34 | Kazakhstan  | 7,26 | 72 | India               | 0,83 |
| 35 | Chile       | 6,53 | 73 | Hong Kong           | 0,82 |
| 36 | Ethiopia    | 6,43 | 74 | Eritrea             | 0,53 |
| 37 | Uzbekistan  | 6,22 | 75 | Trinidad and Tobago | 0,47 |
| 38 | Croatia     | 6,17 |    |                     |      |

There are very few ties in both rankings, mostly due to numerical precision. In the quasi-official ranking all the countries are equally ranked that won only, for instance, one bronze medal. The largest difference between the quasi-official ranking and the second model of our proposed ranking concerned the Czech Republic that went up 14 rungs. The results of those two models are very similar mainly for the first positions.

A comparison between the quasi-official (lexicographic) ranking and the ranking obtained with model 1 shows some differences. The greatest one is Cuba, which was the eleventh country in the lexicographic ranking and drops to the fiftieth position in model 1 ranking. Another country that has a considerable change of rank is China that drops from the second position to the twenty seventh. On the other hand, Argentina that was ranked thirty fourth goes up to fifteenth.

As the quasi-official ranking and the ranking obtained using model 2 are very similar, we expect to obtain almost the same differences between model 1 and model 2. As a matter of fact, when using model 2 China is ranked third and Cuba eleventh and they drop to the previously mentioned positions when using model 1.

#### 4 Final Comments

Two rankings were proposed: one of them takes into account that all sports are equally important. The other one takes into account the “impact” of each sport measured by the number of countries participating in each sport. This measure takes into account as well that the chances to win a medal are not the same in the different sports.

The two indexes obtained have two common characteristics. The first is that they do not overrate gold medals. The second is that they put on an equal footing sports that give away a large number of medals - athletics or gymnastics, for instance - and team sports (such as basket ball or volley ball) in which a single medal rewards a large number of athletes.

The first positions on the ranking using model 2 and according to the quasi-official ranking are almost the same. The major differences appear in the middle and bottom of the table due to other factors including the absence of ties in our method. The similarities of these two rankings can be explained by the approaches being used. In model 2 we used a weighing scheme that takes into account the number of countries disputing medals. The lexicographic method uses the number of medals to rank the countries. It is a fact that in the Olympic Games, the greater the number of medals, the greater the number of participant countries. So, in a way model 2 has a very similar approach than the lexicographic method without over evaluating the gold medal, which might explain the small differences that were found.

The major differences appear when comparing models 1 and 2, i.e., models considering equal and different weights for each sport. As mentioned earlier, China and Cuba go up several rungs as we move from model 1 to model 2. This happens because both countries concentrate their efforts in sports with a large number of medals. This shows an investment in higher “impact” sport. On the other hand, Argentina drops from fifteenth to thirtieth between model 1 and model 2. This shows an opposite strategy, because Argentina concentrates efforts in sports with few medals and few participants such as team sports (football, basketball, and others).

Further developments should include as well the tradition of a given sport in the Olympic Games.

## 5 References

- Anderson, T. R., Hollingworth, K., e Inman, L. (2002) The fixed weighting nature of a cross-evaluation model, *Journal of Productivity Analysis*, Vol 17, pp. 249-255.
- Baimbridge, M. (1998) Outcome uncertainty in sporting competition: The olympic games 1896-1996, *Applied Economics Letters*, Vol 5, No 3, pp. 161-164.
- Balmer, N. J., Nevill, A. M., e Williams, A. M. (2001) Home advantage in the winter olympics (1908-1998), *Journal of Sports Sciences*, Vol 19, No 2, pp. 129-139.
- Balmer, N. J., Nevill, A. M., e Williams, A. M. (2003) Modelling home advantage in the summer olympic games, *Journal of Sports Sciences*, Vol 21, No 6, pp. 469-478.
- Barros, C. P., e Leach, S. (2006) Performance evaluation of the english premier football league with data envelopment analysis, *Applied Economics*, Vol 38, No 12, pp. 1449-1458.
- Bernard, A. B., e Busse, M. R. (2004) Who wins the olympic games: Economic resources and medal totals, *Review of Economics and Statistics*, Vol 86, No 1, pp. 413-417.
- Caloba, G. M., e Lins, M. P. E. (2006) Performance assessment of the soccer teams in brazil using dea, *Pesquisa Operacional*, Vol 26, No 3, pp. 521-536.
- Caporaletti, L. E., Dulá, J. H., e Womer, N. K. (1999) Performance evaluation based on multiple attributes with nonparametric frontiers, *Omega*, Vol 27, No, pp. 637-645.
- Churilov, L., e Flitman, A. (2006) Towards fair ranking of olympics achievements: The case of sydney 2000, *Computers and Operations Research*, Vol 33, No 7, pp. 2057-2082.
- Cooper, W. W., Ruiz, J. L., e Sirvent, I. (2007) Choosing weights from alternative optimal solutions of dual multiplier models in dea, *European Journal of Operational Research*, Vol 180, No, pp. 443-458.
- Doyle, J., e Green, R. H. (1994) Efficiency and cross-efficiency in dea derivations, meanings and uses, *Journal of the Operational Research Society*, Vol 45, No, pp. 567-578.
- Fischer, D. 2003. *The encyclopedia of the summer olympics*: Franklin Watts.
- Foroughia, A. A., e Tamiz, M. (2005) An effective total ranking model for a ranked voting system, *Omega*, Vol 33, No, pp. 491 - 496.
- Haas, D. J. (2003) Productive efficiency of english football teams - a data envelopment analysis approach, *Managerial and Decision Economics*, Vol 24, No 5, pp. 403-410.
- Johnson, D. K. N., e Ali, A. (2004) A tale of two seasons: Participation and medal counts at the summer and winter olympic games, *Social Science Quarterly*, Vol 85, No 4, pp. 974-993.
- Kao, C., e Hung, H.-T. (2007) Management performance: An empirical study of the manufacturing companies in taiwan, *Omega*, Vol 35, No, pp. 152-160.
- Lins, M. P. E., Gomes, E. G., Soares de Mello, J. C. C. B., e Soares de Mello, A. J. R. (2003) Olympic ranking based on a zero sum gains dea model, *European Journal of Operational Research*, Vol 148, No 2, pp. 312-322.

Lozano, S., Villa, G., Guerrero, F., e Cortés, P. (2002) Measuring the performance of nations at the summer olympics using data envelopment analysis, *Journal of the Operational Research Society*, Vol 53, No 5, pp. 501-511.

Mitchell, H., e Stewart, M. F. (2007) A competitive index for international sport, *Applied Economics*, Vol 39, No 5, pp. 587-603.

Morton, R. H. (2002) Who won the sydney 2000 olympics?: An allometric approach, *Journal of the Royal Statistical Society Series D: The Statistician*, Vol 51, No 2, pp. 147-155.

Podinovsky, V. V. (2004) Suitability and redundancy of non-homogeneous weight restrictions for measuring the relative efficiency in dea, *European Journal of Operational Research*, Vol 154, No, pp. 380-395.

Rosen, Schaffnit, C., e Paradi, J. C. (1998) Marginal rates and two dimensional level curves in dea, *Journal of Productivity Analysis*, Vol 9, No 3, pp. 205-232.

Sexton, T. R., Silkman, R. H., e Logan, A. J. 1986. Data envelopment analysis: Critique and extensions. In *Measuring efficiency: An assessment of data envelopment analysis*, edited by Silkman, H. San Francisco: Jossey-Bass Editor.

Soares de Mello, J. C. C. B., Gomes, E. G., Angulo-Meza, L., e Biondi Neto, L. (2008) Cross evaluation using weight restrictions in unitary input dea models: Theoretical aspects and application to olympic games ranking, *WSEAS Transactions on Systems*, Vol 7, No 1, pp. 31-39.

Soares de Mello, J. C. C. B., Lins, M. P. E., e Gomes, E. G. (2002) Construction of a smoothed dea frontier, *Pesquisa Operacional*, Vol 28, No 2, pp. 183-201.

