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ESTATUTO EDITORIAL

«Investigação Operacional», órgão oficial da APDIO cobre uma larga gama de assuntos reflectindo assim a grande diversidade de profissões e interesses dos sócios da Associação, bem como as muitas áreas de aplicação da I. O. O seu objectivo primordial é promover a aplicação do método e técnicas da I.O. aos problemas da Sociedade Portuguesa.
A publicação acolhe contribuições nos campos da metodologia, técnicas, e áreas de aplicação e software de I. O. sendo no entanto dada prioridade a bons casos de estudo de carácter eminentemente prático.

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EDITOR'S NOTE

This number of "Investigação Operacional" is totally dedicated to papers presented at the 21st meeting of the EURO Working Group on Multicriteria Optimization, held in March 1985 in Lisbon.

It has not been usual that these meetings originate a common publication for the papers presented there, just as it has not been usual that this journal dedicates a whole number exclusively to papers from one single meeting or conference; but it has seemed correct to make this sort of "commemorative issue" related to what has been the first of a series of 4 conferences held in Lisbon within about one year, all with the active involvement of APDIO, the Portuguese Association for the Development of Operations Research, the last of those four being EURO VIII, to be held in September 1986.

This series of conferences is a very visible sign of a portuguese approximation effort in direction of the international Operations Research community, where our members are trying to take a more active role in the production of new ideas and in the shaping of decisions. Geographical distance and financial constraints are liabilities we have learnt to live with, against which we will have to play with our energy, initiative and ability to communicate with different peoples.

"Investigaçăo Operacional", the scientific journal of APDIO, is an important tool in this effort: interchange agreements exist with more than 20 national or regional O.R. associations from other countries, and by publishing papers in English, French, Portuguese and Spanish, it can reach a vast catchment area with a great production potential. Indeed by regularly publishing papers from varied countries, we are fulfilling one of our major objectives, namely providing the portuguese technical community (paper authors or others who are "only doing applications") with a wide spectrum of experiences and innovation coming from many different origins.

The papers published here are what has been considered an interesting sample from all the papers presented at the meeting and submitted by the authors. Time limitations have not allowed for a classical refereeing procedure, papers being published as received or with only minor "polishing" in language. For this reason, papers with complex and unclear notation or language have not been included in this issue. It is hoped that the selection made is balanced and the participants in the meeting feel it constitutes a representative and unbiased sample.

The Editor

José Manuel Viegas

COLLECTIVE BARGAINING, TRADE OFF AND CONFLICT ANALYSIS IN MULTIPLE CRITERIA DECISION MAKING

Prof. Dr. W.K. Brauers, Antwerp University

1. THE PROBLEM

Conflict analysis, collective bargaining, trade off etc. are terms connected to group decisions. The human beings belonging to the group like to follow different targets (multiple criteria) for which different solutions (actions) are possible. It is a problem of multiple criteria decision making, trying to come as nearby as possible to the ideal multi-optimal solution.

Going from the more simple to the more sophisticated items, we move facts to value judgments and finally to group interests.

2. MULTIPLE CRITERIA DECISION MAKING ABOUT FACTS

If the multiple criteria existing inside a group concern a misconception about facts, information will reduce the multiple criteria problem to one single criterion: the insight into the common knowledge about facts.

Sometimes information about complicated sỳstems, such as some insight in human behaviour, can be difficult. Systems Analysis, and in particular the several methods of simulation, can be helpful for that ocasion, as we described elsewhere (1).

Next step in this taxonomy concerns the multiple criteria decision making in value judgments.

3. MULTIPLE CRITERIA DECISION MAKING IN VALUE JUDGMENTS

If these value judgments are purely qualificative, no optimality is possible, but if quality can be translated in one or other form of quantification i.e. either cardinal or ordinal, optimality considerations are applicable.

Each individual is sure about his values but uncertain about his knowledge of facts. Each individual has his own prejudices. An individual, for example, may not know who the Prime Minister of Great Britain is but will be 100% sure that communism or fascism is 100% wrong. This means that problems arise for posing broad issues and for obtaining judgmental data. Mostly the knowledge of a group is needed in that case and committees and round-table discussions are not very suitable for the discussion of very broad issues. The main disadvantages have been outlined by E.S. QUADE in the following way:

"In broad problems the range of expertise required is not likely to be provided by a single individual. Almost inevitably a variety of expert advisors needs to be consulted. Experiments have shown that their best use is not the traditional one of having the issues presented to them and debated in open round-table discussion until a consensus emerges or until they arrive at an agreed-upon group position. Committees, for example, often fail to make their assumptions and reasoning explicit. Sometimes the opinions of dissenters are not even recorded. What is needed is a way to avoid the psychological drawbacks of a round-table discussion - such as the "bandwagon" effect and the unwillingness to abandon publicly expressed opinions - and thus to provide a setting in which pros and cons of an issue can be examined systematically and dispassionately" (2)

Moreover, in committees or round table discussions the opinions of very interesting but rather shy persons are often not even heard. A first lesson to be drawn is that the exercise has to be anonymous "pour ne pas perdre la face", for not loosing his face. Check lists of hundreds of items presented to a lot of people are less valuable for broad issues and judgmental data. Who draws up the list for these problems and which people check the lists for occurrence or non-occurrence?

JANTSCH, citing VON FANGE, gives the following basic rules for brainstorming sessions:

"1. State the problem in basic terms, with only one focal point;

2. Do not find fault with, or stop to explore, any idea;

3. Reach for any kind of idea, even if its relevance may seem remote at the time;

4. Provide the support and encouragement which are so necessary to liberate participants from inhibiting attitudes" (3).

In any case an efficient reporting system is necessary to memorize the ideas (stenography or taperecording). Besides this straight brainstorming JANSTSCH cites some variations.

"Whereas straight brainstorming aims primarily at a harvest of new ideas, the "buzz group" technique seeks group consensus among approximately six people.

The operational creativity approach introduces the refinement that only the group leader knows the exact nature of the problem and structures the discussion so as to arrive at a solution - only one is sought" (3).

In the last variation the objectivity of the group leader is doubtful and in general brainstorming is insufficient for tackling broad problems and for obtaining judgmental data. Indeed opinions can be too divergent for a consensus to be reached.

Let us limit ourselves to future development. For future development it depends which universe we are looking after. In a quasi-certain universe techniques such as extrapolation will lead to one single criterion, which is not the case under uncertainty for the future, when so called nominal techniques have to be used.

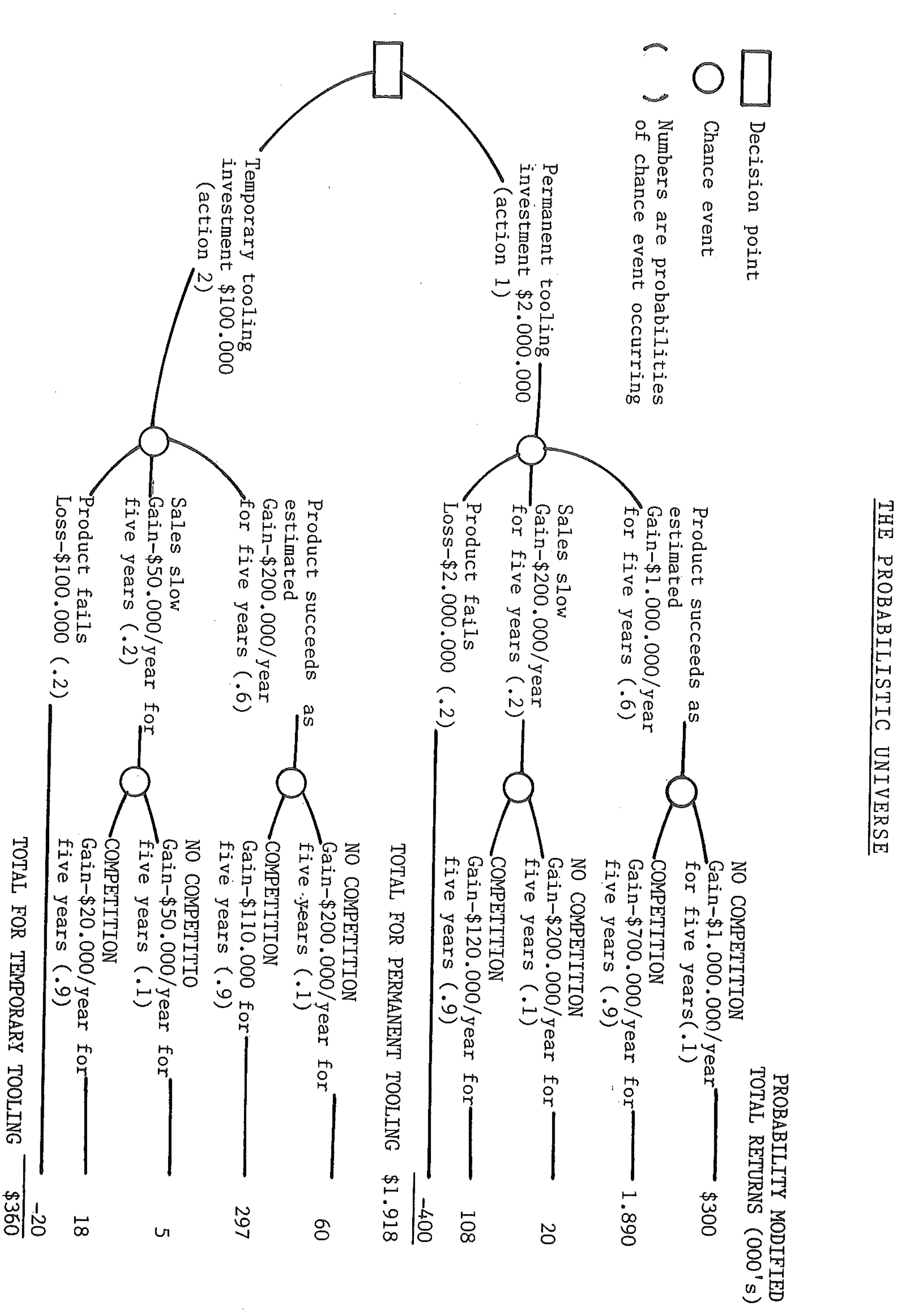
Questionnaires fail if broad issues are involved. Indeed opinions in this case can also be too divergent for a consensus to be reached. The steering group may well make too subjective a summary when analizing the questionnaires. Moreover, questionnaires are considered to be samples for a certain population (polls).

A sample is drawn from the population and the opinion of the sample reflects the opinion of the population. The average of a variable in the sample is considered to be an estimation of the average of the variable for the population concerned.

In this way the "European Coordination Centre for Research and Documentation in Social Sciences" in Vienna interviewed 9,000 persons in ten countries, aged between 15 and 40 at the time of interview, an age group which on the average will reach the year 2,000 (4). More than 50% of those interviewed do not expect a world war before the year 2,000. If the stratified sampling is correct (9,000 persons, 10 countries chosen from all over the world), the majority of the world's population aged between 15 and 40 does not expect a world war before the year 2,000. Besides the cost of the operation, which took three years (1967-1969), a large minority of the age group in question expects a world war, while the question remains unanswered whether there will be a world war or not. The assumption of the steering group that the future state of the world will largely depend on what people want it to be is purely wishful thinking. In this way we do not even know what the probability is of a possible world war before the year 2,000. On such a question there will be no convergence between the opinions of the 9,000 persons. Finally are these persons aged from 15 to 40 able to make a valid judgment on such a question and is this judgment valid for the world's population at large?

Is there not something like expert knowledge? Is a discussion not necessary and how has this discussion to be organized?

The Delphi Technique tries to improve either the panel or


THE PROBABILISTIC UNIVERSE
Decision point
Chance event
( ) Numbers are probabilities of chance event occurring
PROBABILITY MODIFIED TOTAL RETURNS (000's)
Permanent tooling investment $2.000.000 (action 1)
Product succeeds as estimated Gain-$1.000.000/year for five years (.6)
NO COMPETITION Gain-$1.000.000/year for five years(.1)
$300
COMPETITION Gain-$700.000/year for five years (.9)
1.890
Sales slow Gain-$200.000/year for five years (.2)
NO COMPETITION Gain-$200.000/year for five years (.1)
20
COMPETITION Gain-$120.000/year for five years (.9)
108
Product fails Loss-$2.000.000 (.2)
-400
TOTAL FOR PERMANENT TOOLING
$1.918
Temporary tooling investment $100.000 (action 2)
Product succeeds as estimated Gain-$200.000/year for five years (.6)
NO COMPETITION Gain-$200.000/year for five years (.1)
60
COMPETITION Gain-$110.000 for five years (.9)
297
Sales slow Gain-$50.000/year for five years (.2)
NO COMPETITIO Gain-$50.000/year for five years (.1)
5
COMPETITION Gain-$20.000/year for five years (.9)
18
Product fails Loss-$100.000 (.2)
-20
TOTAL FOR TEMPORARY TOOLING
$360

committee, or the questionnaire approach by tacking anonymity and expertise into consideration. Delphi is considered as bringing convergence in opinions. Delphi was so named after the Greek oracle, as it was first thought of as a tool for better forecasting e.g. technological forecasting (5).

How is Delphi introduced into Multiple Criteria Decision Making? It is the case when alternative policies are considered towards alternative futures in probability, possibility or completely uncertain universes.

3.1. Value Judgements in a Probabilistic Universe

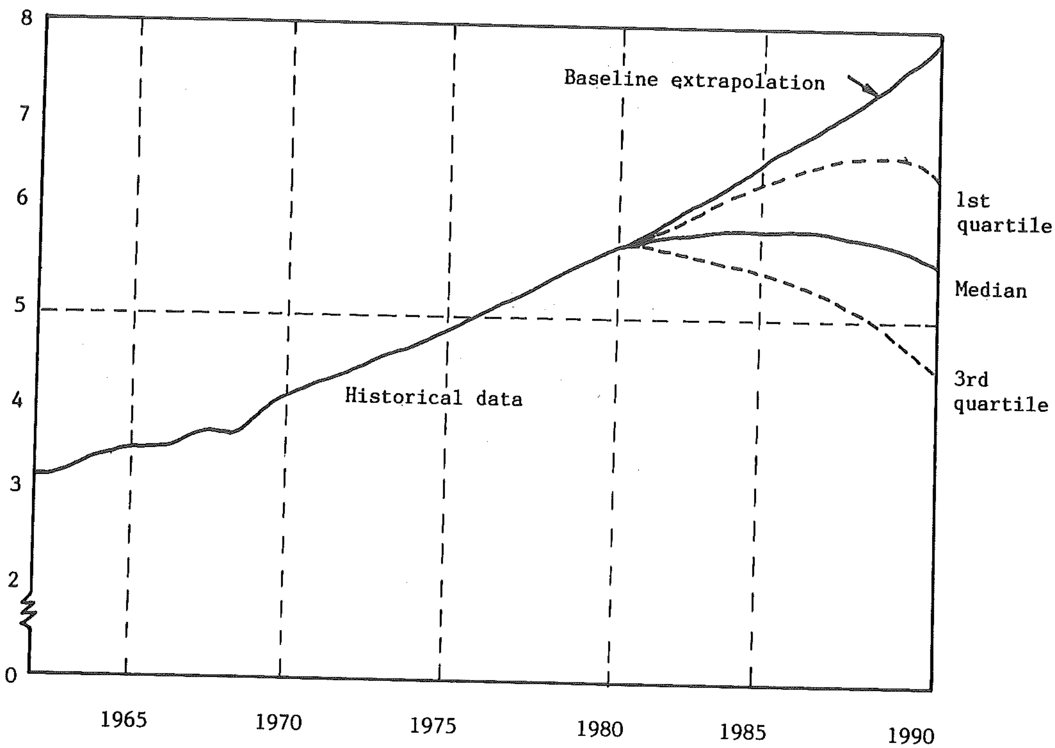
In the following example of a decision tree from the tooling industry, two actions are considered under the following criteria: success of products, slowing down of sales and failure of products, each time under competition and without competition. It concerns the introduction of a new product with the decision to tool up for it in a permanent way with a high investment cost but low manufacturing cost or a temporary tooling in a reversed situation (6).

The probabilities have to be found after several Delphi rounds between experts of the industry, of technology and of marketing /1/.

/1/ It may be recalled that the sum of the probabilities has to be equal to one.

AVERAGE COST OF A PRESCRIPTION (U.S.A.)

(CONSTANT 1970 DOLLARS)



Trend Impact Analysis

3.2. Value Judgements in a Possibility Universe

3.2.1. The example of trend impact analysis

In this case of trend impact analysis several actions or policies are considered which may maintain or change the slope to a certain extent of the historical trend (see next graph). The possibility-ratios are once again found in a delphi-exercise /2/.

A unique solution is found by considering as much by considering as much actions or policies as possible and then by taking the median.

3.2.2. The example of Cross-Impact Analysis

In cross-impact analysis several actions or policies are matched against several desired events by possibility-ratios found in a delphi-exercise (see next graph).

Solution is possible either voluntaristic or probabilistic.

Voluntaristic means that thresholds are put e.g. desired event II has to have a probability of occurrence of at least 0.60. At that moment only policy B is possible.

/2/ In a possibility universe the possibility-ratios are not adding up to one. Probabilities are then the sum of the occurrence plus the non-occurrence of an event.

POSSIBILITY UNIVERSE AND CROSS-IMPACT

CRITERIA ACTIONS	DESIRED EVENT I	DESIRED EVENT II	DESIRED EVENT III
POLICY A	$P_0 = 0.60$	$P_0 = 0.50$	$P_0 = 0.60$
POLICY B	$P_0 = 0.50$	$P_0 = 0.60$	$P_0 = 0.60$

If the solution may be probabilistic, we proposed elsewhere an algorithm for solving the cross-impact problem (1).

The multiple criteria decision making becomes much more difficult if it concerns personal and commom interests.

Let us call this application "multiple criteria decision making about group benefits".

4. MULTIPLE CRITERIA DECISION MAKING ABOUT GROUP BENEFITS

Under this heading a study is made about group versus personal benefits. In this connection the term "group" covers a variety of meanings from small groups of persons and pressure groups to net social benefits - being the difference between total social benefits and social costs - and external economies. Once again, optimization is aimed at.

4.1. The Pareto-optimum

Generally one goes out from the so-called Pareto-optimum in order to have an insight in this optimization problem. We thought it useful to go back to Pareto in order to see what he exactly meant by this optimum. He says that the collective maximum of utility is reached when no departure from this position is possible without harming one individual to the benefit of another /1/.

/1/ Pareto rather speaks of "ophélimité" instead of "utility". "L' Ophélimité ... consiste dans le plaisir que procure la quantité totale de la marchandise A possédée". (V. PARETO, Manuel d' Economie Politique, traduit sur l' edition italienne par A. BONNET, revue par l' auteur, deuxième éd. Paris 1927, p. 263). On p. 169 n.1 Pareto however puts "ophélimité" equal to "utilité".

He defines his famous optimum as: "Nous dirons que les membres d' une collectivité jouissent, dans une certaine position, du maximum d' ophélimité, quand il est impossible de trouver um moyen de s'éloigner très peu de cette position, de telle sorte que l' ophélimité dont jouit chacun des individus de cette collectivité augmente ou diminue. C' est-à-dire que tout petit déplacement à partir de cette position a nécessairement pour effect d' augmenter l' ophélimité dont jouissent certains individus, et de diminuer celle dont jouissent d' autres: d'être agréable aux uns, désagréable aux autres.
(V.PARETO, op. cit. p. 354).

Pareto proved that free competition on the market leads to this optimum /1/. As all the so-called classical authors in economics, he defines free competition as the situation in which neither the suppliers nor the demanders can influence the price on the market /2/.

The optimum is not automatically reached after Pareto, if this market mechanism is absent. It is so in the following cases:

1) in the monopoly situation, where the monopolist himself will fix prices and quantities beneficial for himself but detrimental for the others /3/;

/1/ V.PARETO, op.cit. pp. 354-361.

/2/ "Le type (I) des phénomènes est donné par ceux dans lesquels l'individu accepte les prix qu'il trouve sur le marché et cherche à satisfaire ses gouts avec ces prix. En ce faisant, il contribue, sans le vouloir, à modifier ces prix, mais il n'agit pas directement dans l'intention de les modifier. A un certain prix il achète (ou il vend) une certaine quantité de marchandise; si la personne avec laquelle il contracte acceptait un autre prix, il achèterait (ou il vendrait) une autre quantité de marchandise. En d'autres termes, pour lui faire acheter (ou vendre) une certaine quantité de marchandise, il faut pratiquer un certain prix." (op.cit. p. 209)

....

"Nous avons déjà vu que, dans la réalité, le type (I) correspond à la libre concurrence et que le type (II) correspond au monopole." (op.cit. p. 210)

/3/ V.PARETO, op.cit. p. 210 and p.356

2) in the collectivistic society where the state will fix prices in order to reach the optimum /1/;

3) in the case of all kind of pressure groups /2/;

/1/ "le type (III) correspond lui aussi au monopole; mais il se distingue du type (II) par le but qu' il se propose. Le problème que devra se poser l' Etat socialiste est le suivant; "Quel prix dois-je fixer pour que mes administrés jouissent du bien-être maximum compatible avec les conditions dans lesquelles ils se trouvent, ou que je trouve bon de leur imposer?" (op.cit. pp. 210-211).

Under the heading "l'équilibre dans la société collectiviste" Pareto discusses the problems in relation to the optimum in the collectivistic society (pp. 362-364).

/2/ "En réalité, les syndicates ouvriers, les producteurs qui jouissent de la protection douanière, les syndicats de négociants qui exploitent les consommateurs, nous fournissent de nombreux exemples dans lesquels les coefficients de production sont déterminés dans le but de favoriser certaines collectivités partielles.

Il faut remarquer que, sauf certains cas tout à fait exceptionnels, ces valeurs de coefficients diffèrent, et souvent diffèrent beaucoup des valeurs qui procurent le maximum d' ophélimité à la collectivité tout entière. (op.cit. p.365)

It seems that Pareto was very prophetic for the actual situation as his original work of the "Manuel d´ Economie Politique" was written in Italian in 1906. In the line of Pareto´ s thoughts problems of optimization arise for nonmarket goods and services, also called social or collective goods and for the human being in a rationalized economic system. It is also possible that group benefits are larger than the sum of the corresponding individual benefits. It is the case with external economies and with synergy effects (7). An application of the Pareto-optimum in a transitive way concerns the rules of the "General Agreement on Tariffs and Trade", authorizing customs unions on condition that the common tariff does not exceed the average tariffs of the potential member countries (8). A more qualitative application is the federalization approach, as launched by H.BRUGMANS, stating that divergent ideas and principles have to be federalized more or less after the greatest common divisor (9).

The Pareto optimum however forms a small basis for any activity; in fact its application would mean immobilism instead of action. In an active policy one can´t exclude winners opposite losers.

4.2. Optima away from the Pareto-optimum

The KALDOR - HICKS test states that a situation is esteemed better than a previous one, if the sum of the benefits of the winners is larger than the sum of the losses of the losers (10).

The VINER-principle in international trade, stating that in forming a customs-union the total trade creation has to be larger than the total trade diversion (11), is an application of the Kaldor - Hicks test.

Unlimited application of the test in politics leads to despotism towards minory groups. This is certainly disastrous, when a dictator, like Hitler or Stalin, determines the total benefit for the society.

4.2.1. The treshold method

An answer to this extreme situation is given by a nontransitive approach, in which tresholds of human rights are fixed in order to protect the individual. In a democracy laws are the result of the majority vote, but the treshold is formed by the constitution which in the first place is a declaration of human rights, protecting each individual.

4.2.2. The Nominal Group Techniques

The Nominal Group Techniques as cited above, such as delphi, trend impact and cross impact analysis, can be useful as applied in a policy of group benefits, creating more convergence between extreme positions, contrary to meetings and polls.

It is certain that a lot of firms in the United States and in Western Europe were using delphi techniques to test several strategies based on divergent interests or to apply it in participatory planning, though no communication is made to the outside world. The first known application is made at T.R.W. of the Hughes-group in California (1969), while Agfa--Gevaert, Belgium was also active in this field (12).

4.2.3. Antagonistic Criteria

An example of antagonistic criteria is the "Ooster-Schelde" problem. In 1953 the islands of the province of

Zeeland in the Netherlands were flooded causing the death of thousands and thousands of persons and billions fl. of material losses. Closing the islands with one huge dam, making the islands a part of the continent and changing the "Ooster-Schelde" estuarium in a huge sweet water reservoir presented a good solution for the security people but was found very harmful by the ecologists. Higher dikes on the islands were accepted by the ecologists but not by the safety people. In this way the "Ooster-Schelde" problem was a good example of antagonistic criteria defended by several groups.

The breakthrough would come by finding a solution acceptable for the parties and more or less satisfying the criteria. It means an effort of creative thinking. Brainstorming as cited above may be useful in this context, but also the Scorecard method. The Scorecard method was invented by Rand Corporation in the case of the Ooster-Schelde (13). All the advantages and disadvantages of all the propositions were enumerated in a systematic way by tables, graphs etc. On basis of this information, new solutions were looked after bringing a kind of greatest possible divisor for all criteria. The solution found consisted of storm dams in front of the islands which would weaken the floods, but keep the Ooster-Schelde as an open estuarium with salt water. This solution was satisfactory to both security officers and ecologists, and the proposition finally passed in Dutch Parliament.

Perhaps one criterion was however overlooked at that time viz. the increase in costprice of such gigantic hydraulic public works.

We tried to bring a taxonomy about group decisions under multiple criteria. We do not know if we were complete. We hope to have many reactions in order to make this taxonomy much consistent and complete.

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DEUX PETITES METHODES MULTICRITERES D'USAGE COURANT

Alain Schärlig
HEC Lausanne

Pratiquer le multicritère, c'est aussi produire - ou proposer - des méthodes d'usage courant, notamment quand le recours aux grands raisonnements est exclu du fait de l'interlocuteur (qui ne veut pas, ou ne peut pas, utiliser une méthode élaborée). Deux exemples récents méritent d'être racontés aux membres du Groupe européen.

OFFICE DE PLACEMENT, AGENCE MATRIMONIALE, MÊME COMBAT!

La première application se situe dans un office de placement, qui met en relation des employeurs potentiels et des demandeurs d'emploi. Cet office désirait automatiser la consultation de son fichier des candidats. L'informatisation n'a posé qu'un problème: on s'est rendu compte que la comparaison, entre les exigences de l'employeur et les caractéristiques des candidats, avait été faite jusqu'ici avec l'esprit de synthèse et de flou dont est capable le cerveau humain ... ce qu'une machine a beaucoup plus de peine à réaliser.

La question a été traitée en multicritère. On a divisé les exigences de l'employeur - les critères - en trois catégories: l'impératif, dont l'absence entraîne obligatoirement l'élimination du candidat; le hautement souhaitable, dont l'absence chez le candidat seraît gênante, mais sans entraîner nécessairement son élimination; et enfin le souhaitable, dont la présence augmenterait l'intérêt porté au candidat. Au moment du passage en revue d'une fiche, on utilise un premier crible oui-ou-non, qui ne rétient que les candidats présentant les caractères impératifs. Sur ces candidats, on compte pour chacun le nombre de caractères hautement désirables qu'ils présentent, et ce nombre sert à opérer un classement, du "meilleur" candidat au "moins bon". Enfin, toujours au moment du passage en revue, on note les caractéristiques simplement désirables présentées par chaque candidat, et ce compte-là permet de départager les ex-aequo du classement précédent. Les candidats sont alors présentés à l'employeur potentiel dans l'ordre donné par ce classement.

L'application, qui tourne à Genéve, a été vendue à une agence matrimoniale de Francfort : du point de vue multicritère, le problème est le même!

ABRIS DE PROTECTION CIVILE : UNE SOMME BIAISÉE.

La Suisse se préoccupe de mettre sa population civile à l'abri en cas de guerre (conventionelle ou nucléaire). Les abris sont longs et chers à construire (la couverture actuelle est de 70% de la population), ce qui fait que certaines communes sont en déficit. Pour le cas où un conflict se présenterait à court terme, elles doivent donc prévoir des "abris de fortune", qui seraient mieux que rien. Encore faut-il être certain qu'on puisse y mettre des gens à l'abri sans accroîte les risques qu'on leur fait prendre. Comme ces risques peuvent être de diverses espèces, car on ne sait pas à l'avance lequel se présentera, le problème est multicritère : les abris de fortune doivent présenter une bonne protection vis-à-vis de plusieurs risques différents.

On a cherché une méthode d'évaluation simple, qui puisse être pratiquée sans difficulté par un responsable local de la Protection civile. La formule proposée par l'Office fédéral de la protection civile recourt à la somme de notes - dont on connaît bien les défauts dans le Groupe - mais en corrigeant le plus gros de ces défauts qui est la compensation d'une mauvaise note par une bonne : il ne faudrait pas, en effet, retenir un abri parce qu'il est une excellente protection contre certains risques, alors qu'il serait mauvais vis-à-vis d'autres risques. Pour ce faire, l'échelle des notes retenues a été :

très bon	1
bon	2
utilisable	5
mauvais	40.

Les critères étant au nombre de 10, et l'addition ne faisant l'objet d'aucune pondération, on applique ensuite une échelle très restrictive pour le jugement global :

10 ou 11 points	très bon
12 à 20 points	bon
21 à 50 points	utilisable
plus de 50 points	mauvais.

On voit qu'en donnant la note 40 pour "mauvais", on rend la somme non compensatoire : un abri réputé mauvais à un seul point de vue ne peut être déclaré globalement ni très bon ni bon, et se trouve même à la limite entre l'utilisable et le mauvais s'il a été très bon à tous les autres points de vue (son total étant alors de 49).

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A NOTE ON THE PAY-OFF MATRIX IN MULTIPLE OBJECTIVE PROGRAMMING

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ABSTRACT

The pay-off matrix is a well-known device in multiple objective programming. It helps to understand the conflicts between different goal variables. Furthermore, the pay-off matrix constitutes the basis for the calculation of frequently used references vectors such as the ideal (utopia) vector and the nadir vector. Unfortunately, when the separate optimization of the individual goal variables does not result in unique optimal solutions, the corresponding nadir vector is not uniquely defined.Ignoring this phenomenon may have a serious effect on several multiple objective programming methods. In this paper we re(de)fine the notion of nadir vector in order to take account of the possibility of alternative optimal solutions. We also present a procedure which generates this uniquely defined nadir vector.

Acknowledgment

We would like to thank professor Lootsma for his valuable comments.

1. INTRODUCTION

One of the key concepts in multiple objective optimization is the pay-off matrix. The elements of the pay-off matrix P are defined as : $p_{ij} = g_i(x^{-j})$, i, j = 1, ..., m, where $g_i(x)$, i = 1,...,m are the goal variables to be maximized as functions of the instruments x, and x^{-j} denotes the instrument vector which minimizes the j-th goal variable within the feasible region K. The pay-off matrix is a valuable tool in investigating the conflicts between the goals of the decision problem at hand. Two important vectors which can be derived from this matrix are the ideal (utopia) vector $g^*=(g^*_1, \ldots, g^*_m)$, with :

$$g^*_j = g_j(x^{-j}), \; j = 1, \ldots, m$$

and the nadir vector $n=(n_1, \ldots, n_m)$ with :

$$n_j = \max_{i=1,\ldots,m} g_j(x^{-i}), \; j=1, \ldots, m.$$

For example, given the pay-off matrix

	Maximize g_1	g_2	g_3	g_4
g_1	7	9	8	10
g_2	9	3	15	20
g_3	6	4	1	4
g_4	100	95	50	10

the ideal vector (g^*) and nadir vector (n) can be determined as :

$$g^* = (7,3,1,10)$$

and

$$n = (10,20,6,100).$$

Both the ideal and the nadir vector are frequently used in interactive methods (e.g. in STEM-type methods).

An often overlooked problem is that the pay-off matrix is not necessarily unique because different x^{-j} may produce the same g^*_j value. As a consequence, the nadir vector is not necessarily unique either. In this paper we address this problem by first presenting two examples with non-unique nadir vectors (Section 2). Next, we re(de)fine the concept of the nadir vector in Section 3. A procedure to determine the nadir vector is given in Section 4. In the final section we discuss the computational complexity of the proposed procedure.

2. NON-UNIQUE PAY-OFF MATRICES

The possible implications of a non-unique pay-off matrix for the calculation of the nadir vector can best be demonstrated by means of two simple examples. In both examples there is a set of possible nadir vectors, only one of which is the "true" one. In this section we will use intuition to determine the "true" nadir vector. In the following section this notion will be more precisely defined. The first example shows that a randomly chosen nadir vector may have higher values than the true nadir vector (assuming all goal variables are to be minimized).

In the second example we show that a randomly chosen nadir vector may have lower values than the true nadir vector (again assuming all goal variables are to be minimized).

Example 1

In Figure 1, the set of feasible solutions of a given multiple objective programming is represented in goal value space . Clearly, there is only one solution (B) yielding the optimal goal value g^*_2. In contrast, all solutions on the line segment (C_1, C_2) are optimal with respect to $g_1(x)$. If no attention would be paid to the non-uniqueness of

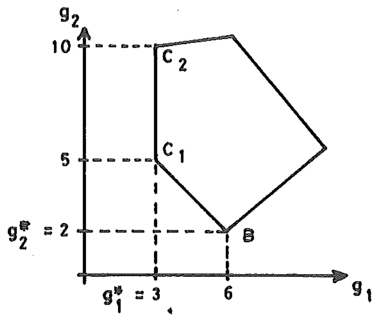

FIG. 1

the optimum for goal variable $g_1(x)$ different pay-off matrices and, consequentely, different nadir vectors might result. In this case, we might have as pay-off matrix either

$$P_1 = \begin{pmatrix} 3 & 6 \\ 5 & 2 \end{pmatrix} \quad \text{or } P_2 = \begin{pmatrix} 3 & 6 \\ 10 & 2 \end{pmatrix}$$

with corresponding nadir vectors

$$n_1 = (6,5) \quad \text{or} \quad n_2 = (6,10).$$

The fact that C_1 dominates all alternative solutions on the line segment (C_1, C_2) is an intuitive ground to define n_1 as the "true" nadir vector. In order words, if the nadir values would be imposed as constraints (e.g. in STEM-type methods), the n_1-values would exclude a larger number of inferior (dominated) solutions from the remaining set of feasible solutions than all other possible nadir vectors. At the same time, the n_1-values would not exclude any efficent solution.

Example 2

The second example concerns a problem described in Kok, 1984. In this problem, each of three goal variables is to be minimized. The optimum of the first goal variable is unique whereas the optimum of both other goal variables is non-unique. Below we summarize the alternative goal vectors (corner solutions) yielding the optimal goal values.

	g^*_1	g^*_2		g^*_3			
g_1	28.75	30	30.5	29.375	32.75	28.75	32.5
g_2	250	5	5	225.5	120.5	250	133.3
g_3	0	50	45	0	0	0	0

On basis of these solutions, eight different pay-off matrices can be constructed, resulting in six different nadir vectors. Apart, from the goal vector (32.5,133.3,0), all alternative solutions are efficient. The "true" nadir vector would (intuitively) be defined as the vector of maximum values obtained within the given set of efficient goal vectors. That is, the true nadir vector would be :

$$n_T = (32.75,250,50).$$

Notice that if any of the other goal vectors would be selected as nadir vectors and if the values of such a nadir vector would be imposed as constraints, a subset of the efficient set would be excluded. For instance, the values of the potential nadir vector (32.75,250,45) would, if imposed as constraints, exclude the goal vector (30,5,50) which is nonetheless an efficient solution.

3. CELLAR, NADIR AND PESSIMISTIC VECTORS

To solve the problems sketched above, we need a more precise definition of the nadir vector concept. In the remainder of the paper we will assume that all goal variables $g_i(x)$, $i=1,\dots,m$, are to be maximized within the feasible region K. The efficient set of decision problem is denoted by E. In addition, we define $K_i=\{x|x\in K \wedge g_i(x)=g^*_i\}$, with $g^*_i = \max_{x\in K} g_i(x)$.

Furthermore, let $K_{j|i}=\{x|x\in K_i \wedge g_j(x)=g^*_{j|i}\}$ with $g^*_{j|i} = \max_{x\in K_i} g_j(x)$;

$K_{k|j|i}=\{x|x\in K_{j|i} \wedge g_k(x)=g^*_{k|j|i}\}$ with $g^*_{k|j|i} = \max_{x\in K_{j|i}} g_j(x)$; etc.

Next, we define the nadir vector n as the vector of goal values of which the j-th element is given by :

$$n_j = \min_{x\in \bar{K}} \{g_j(x)\},$$

with $$\bar{K} = \bigcup_{\substack{i_1 = 1, \dots, m \\ \vdots \\ i_m = 1, \dots, m}} K_{i_1|\dots|i_m}$$

Note that this nadir vector is unique.

It is often assumed that the elements of the nadir vector represent the minimal values of the goal variables over the entire efficient set. In many cases this is not correct (even not with the precise definition of the nadir vector), as is shown by Dessouki et.al., 1979, Spronk, 1981, and Weistroffer, 1983. Therefore, we define the cellar vector c as the vector of which the elements are the minimal values of the goal variables over the entire efficient set:

$$c_j = \min_{x \in E} \{g_j(x)\}, \; j=1, \ldots, m.$$

As mentioned above, for the redefined nadir vector $n_j \geq c_j$ for $j=1,\ldots,m$ in the general vector maximization problem.

The elements of the nadir and cellar vector can be seen as parameters of the decision problem at hand, which - as such - have nothing to do with the preferences of the decision--maker. For the case the decision-maker has defined a series of minimimally required goal values, we propose to use the term pessimistic vector. The elements of the pessimistic vector p are the lowest goal values which the decision-maker considers to be acceptable. Obviously, the decision-maker may choose any (feasible) value of p_j. That is, he may choose $p_j \leq c_j$, $c_j \leq p_j \leq n_j$, or $p_j \geq n_j$. If the decision-maker does not exactly know what he wants, one should be careful in defining the p_j-values for him. In well-defined models, it is reasonable to choose $p_j = c_j$, $j = 1,\ldots,m$. However, by choosing $p_j \geq n_J$, $j = 1,\ldots,m$ (with n an arbitrary nadir vector) one runs the risk of excluding efficient solutions.

4 . DETERMINATION OF THE NADIR VECTOR

The definition of the nadir vector suggests that it is necessary to solve a great number of optimization problems. However, if the solution of one of these problems - e.g. $\max_{x \in K} g_j(x)$ - is unique, then it is immediately clear that this solution is the only feasible vector in $K_{i_1 | \ldots | i_{m-1} | j}, i_1 \neq j, \ldots, i_{m-1} \neq j$. This means that a test, whether the solution of the optimization problem at hand is unique or not, may reduce the number of optimization problems considerably.

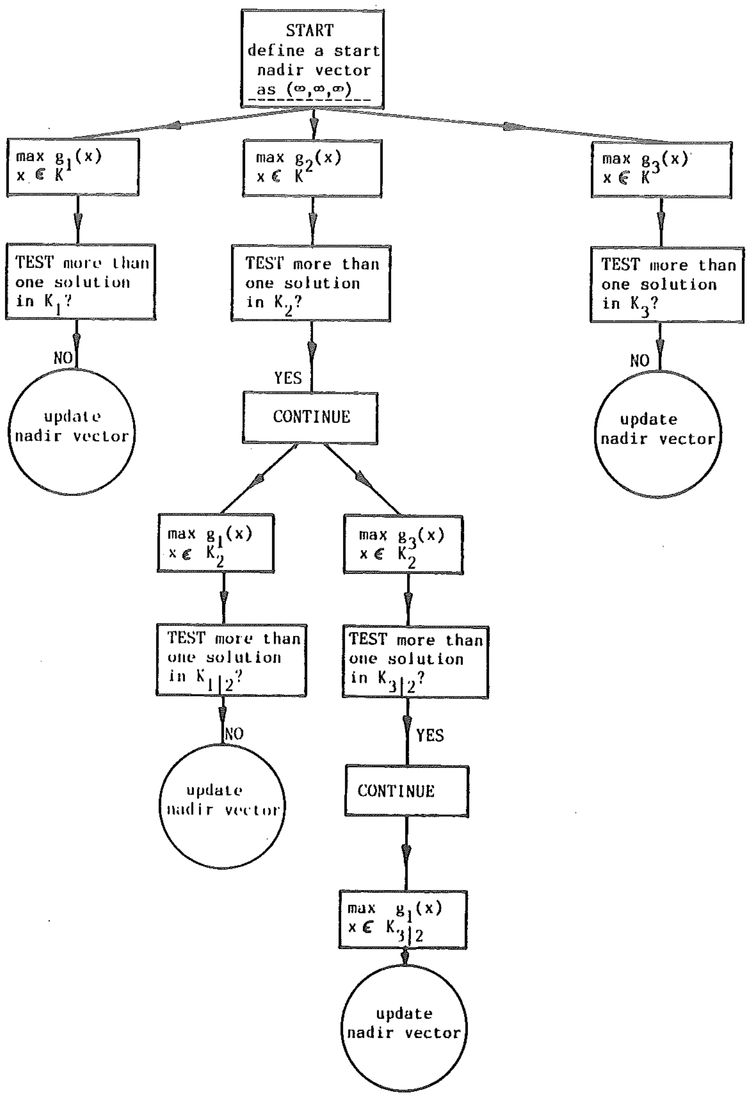

Figure 2

To avoid unnecessary technical details, we will not give a formal presentation of the procedure to determine the nadir vector. Instead, we illustrate the procedure by means of a simple decision problem with three goal variables $g_1(x)$, $g_2(x)$ and $g_3(x)$. The elaboration of the procedure in this simple case is summarized in Figure 2. First, we set all elements of the nadir vector equal to infinity. Then, the first goal variable, $g_1(x)$, is optimized within the set of alternatives, K, and we test whether the solution of this optimization problem is unique. In this example, the solution is unique. The nadir vector is updated by taking as its elements the values of the goal variables in this unique solution. Then the goal variable $g_2(x)$ is optimized. Now we have alternative solutions, by which it becomes necessary to perform another two optimizations. From the flow-chart in Figure 2 it can be concluded that the solution of $\max_{x \in K_2} g_1(x)$ is unique.

Therefore, the nadir vector must be updated by adopting those goal values resulting from $\max_{x \in K_2} g_1(x)$ which are lower than the corresponding values in the old nadir vector. The solution of $\max_{x \in K_2} g_3$ is not unique, so we have to perform another optimization : $\max_{x \in K_{3|2}} g_1(x)$, etc.

Finally, we add a technical remark concerning the implementation of the procedure. The test whether there are alternative solutions or not can be carried out with the reduced-costs vector : if at least one element of this vector related with the non-basic variables (in linear programming the final row 0 in the Dantzig Simplex tableau, see e.g. Wagner, 1975) is the zero coefficient, we have alternative optima.

5. DISCUSSION

In this section we discuss some properties of the procedure to determine the redefined nadir vector. First, we discuss the computational complexity of the procedure. It can easily be verified that the structure of the proposed procedure is a tree, where the nodes correspond to the optimization problems. The maximum number of nodes n(m), with m the number of goal variables, is :

$$n(m) = \sum_{p=0}^{m-1} \frac{m!}{P!}.$$

The values of n(1), ..., n(6) are given in table 1.

m	n(m)
1	1
2	4
3	15
4	64
5	325
6	1956

Table 1

Of course, the formula given above represents certainly the worst case : all optimization problems in the procedure have always alternative optima. In general, only some of the optimization problems will have alternative optima, limiting the number of problems to be optimized. Second, as mentioned above, the redefined nadir vector is not equal to the cellar vector (Section 3). Nevertheless, the nadir vector as defined here may constitute a good starting point to find elements of the cellar vector. Of course, other starting points might be appropriate. However, the question which is the best one cannot be answered before a good procedure to find the cellar vector is available.

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UTASTAR : AN ORDINAL REGRESSION METHOD FOR BUILDING ADDITIVE VALUE FUNCTIONS

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ABSTRACT

This paper presents an improved version of the UTA method performing an ordinal regression analysis by means of more powerful linear programming formulations. The ordinal variable to be analysed is a weak-order relation whereas the independent variables are criteria, i.e. quantitative and/or qualitative monotone variables. The method is illustrated by a simple numerical example. Finally, experimental results are given, demonstrating by means of three distinct indicators, the superiority of the adjustements obtained with the new method.

Key-words : Ordinal Regression; Additive Utility; Multicriteria Analysis.

1. INTRODUCTION

The problem of ordinal regression which is presently dealt with the UTA method of JACQUET-LAGREZE and SISKOS (2) is the following : Having a weak-order preference structure $(\succ, \sim)$, with the strict preference and $\succ$ the indifference on a set of actions or objects, adjust additive utility functions based on multiple criteria in such a way that the resulting preference structure would be as consistent as possible with the initial structure.

Let $A = \{a, b, c, \ldots\}$ be the set of actions upon which the preference structure is given. Let $g_1, g_2, \ldots, g_n$ be a family of n criteria. Each one is defined here, under the form of a real valued monotone function $g_i : A \rightarrow [g_{i*}, g^{i*}] \subset R$ in such a way that $g_i(a)$, $a \in A$ represents the evaluation of the action a on the criterion g_i and g_{i*}, g_i^* respectively the level of the most and the least desirable of the criterion.

When only one criterion is concerned, the preferences can be explicited as following :

$$a \succ b \iff g_i(a) > g_i(b) \quad (1)$$

$$a \sim b \iff g_i(a) = g_i(b) \quad (2)$$

which means that each criterion defines on the set A a weak--order relation $(\succ, \sim)$.

A utility function under certainty is a real valued function u :

$$\overset{n}{\underset{i=1}{X}} \left[g_{i*}, g_i^*\right] \rightarrow R \text{ such that}$$

$$a \succ b \Longleftrightarrow u \left[\underline{g}(a)\right] > u \left[\underline{g}(b)\right] \quad (3)$$

$$a \sim b \Longleftrightarrow u \left[\underline{g}(a)\right] = u \left[\underline{g}(b)\right] \quad (4)$$

where $\underline{g}(a) = \left[g_1(a), g_2(a), \ldots, g_n(a)\right]$ is the profile of the action a consequences on the n criteria.

The UTA regression aims to estimate additive utilities :

$$u(\underline{g}) + u_1(g_1) + u_2(g_2) + \ldots + u_n(g_n) \quad (5)$$

satisfying

$$u_i(g_{i*}) = 0 \ \forall \ i \quad (6)$$

$$u_1(g_1^*) + u_2(g_2^*) + \ldots u_n(g_n^*) = 1 \quad (7)$$

So, the relations (5)-(7) normalize the marginal utilities u_i and the total utility u between 0 and 1.

In the original version of UTA method, there exists a unique error function $\sigma : A \rightarrow [0,1]$ where $\sigma(a)$ is the amount of utility which would be suitable to add to the estimated utility $u\left[\underline{g}(a)\right]$ of the a action in order to make it possible for this action to regain its rank in the weak-order (fig. 1). This error function is not sufficient to minimize completely the dispersion of points all around the monotone curve of figure 1. The problem is posed by points situated on the right of the curve, from which it would be suitable to substract an amount of utility and not increase the utilities of the others.

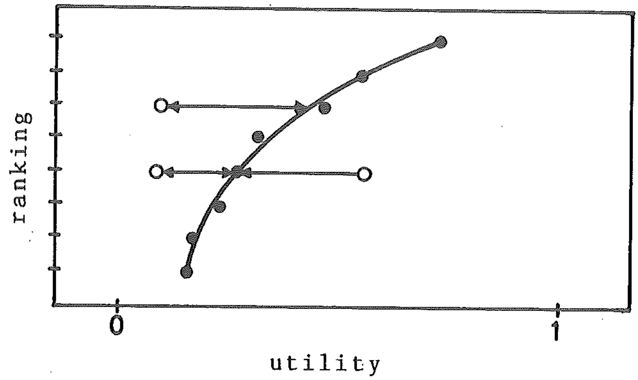

Fig. 1

Utility versus weak order in ordinal regression

In the version of UTA proposed in this paper (let us call it UTA*), we use a double positive error function which permits to better stabilize the position of the points around the curve.

So, the utility of an action a will be introduced into the relations (3)-(4) through $u\,[\underline{g}(a)] + \sigma^{+}(a) - \sigma^{-}(a)$.

In the following section we present briefly the UTA model as well as its improvements. Section 3 presents an illustration of UTA*, based on a numerical example with five alternatives and three criteria. In the last section we show the superiority of the new model with respect to the old one, based on three different indicators.

2. UTA AND UTA*

UTA uses a special linear programming formulation to estimate the marginal utilities u_i under the conditions (5)-

(7). This estimation is made after having discretized each interval of varying criteria

$$[g_{i*}, g_i^*] = [g_{i*} \equiv g_i^1, \ldots, g_i^{\alpha_i} \equiv g_i] \qquad (8)$$

and introduces the constraints $u_i(g_i^{j+1}) \geq u_i(g_i^j)$ in order to preserve the monotonicity of the criterion. The number of the equally distant point α_i is calculated by an algorithm as far as information is available. For the quantitative criteria we have employed the technique of linear interpolation.

According to conditions (5)-(7), the original UTA algorithm runs in four steps :

1) Expression in the order imposed by the initial weak-order $(\succ, \sim)$ the utilities of the alternatives $u[\underline{g}(a)]$, $a \in A$ in terms of additive marginal utilities $u_i(g_i^j)$.

2) Going from head to tail of the weak-order by writing for each pair (a,b) of consecutive actions, the analytic expressions:

$$\Delta(a,b) = u[\underline{g}(a)] - u[\underline{g}(b)] + \sigma(a) - \sigma(b) \qquad (9)$$

The number of these expressions is equal to the number of actions minus 1.

3) Solving the dual of the linear programme :

$$\text{Minimize } F = \sum_{a \in A} \sigma(a)$$

under the constraints (according to step 2)

$$\Delta(a,b) \geq \delta \text{ if } a \succ b$$
$$\Delta(a,b) = 0 \text{ if } a \sim b$$

$$u_i(g_i^{j+1}) - u_i(g_i^{j}) \geq 0 \ \forall \ i \text{ and } j$$

$$\sum_i u_i(g_i^{*}) = 1$$

$$u_i(g_i^{1}) = 0,\ u_i(g_i^{j}) \geq 0,\ \sigma(a) \geq 0 \ \forall \ a \in A,\ \forall \ i \text{ and } j$$

δ a small positive value.

4) Testing the existence of multiple or near optimal solutions (stability analysis). In case of non uniqueness, find those optimal solutions which maximize the "weights" $u_i(g_i^{*}) \equiv u_i(g_i^{\alpha_i})$ for each i.

The modifications integrated into the new model are, by step, the following :

1) The monotonicity constraints of criteria are taken into account in the transformations of variables

$$w_{ij} = u_i(g_i^{j+1}) - u_i(g_i^{j}) \geq 0 \ \forall \ i \text{ and } j. \tag{10}$$

The utilities u $[\underline{g}(a)]$ are becoming functions of w_{ij}, i.e. as

$u_i(g_i^{1}) = 0$, for j > 1 we have

$$u_i(g_i^{1}) = \sum_{k=1}^{j-1} w_{ik} \tag{11}$$

2) Introduction of a double error function: Write for each pair of consecutive actions (a,b) of the weak-order

$$\Delta(a,b) = u\,[g(a)] - u\,[g(b)] + \sigma^+(a) - \sigma^-(a) - \sigma^+(b) + \sigma^-(b) \quad (12)$$

3) Solving the primal of the linear programme :

$$\text{Minimize } F = \sum_{a \in A} \{\sigma^+(a) + \sigma^-(a)\}$$

under the constraints

$$\Delta\,(a,b) \geq \delta \quad \text{if } a \succ b$$

$$\Delta\,(a,b) = 0 \quad \text{if } a \sim b$$

$$\sum_i \sum_j w_{ij} = 1$$

$$w_{ij} \geq 0,\ \sigma^+(a) \geq 0,\ \sigma^-(a) \geq 0\ \forall\, a \in A,\ \forall\, i \text{ and } j$$

δ a small positive value.

4) No change. The "weights" become : $u_i(g_i^*) = \sum_{k=1}^{\alpha_i - 1} w_{ik}$.

Remark : This formulation is similar to those developed in goal programming [1] . It is so easy to prove that, in the optimum, we have $\sigma^+(a) \cdot \sigma^-(a) = 0\ \forall\, a \in A$, i.e. at least one error per action is nil.

3. A NUMERICAL EXAMPLE

Let us consider the case of an individual whose choice of transportation means home-work place during the peak hours is analysed. Our decision-maker is interested only in three criteria (1) price (in French Francs), (2) time of journey (in minutes) and (3) comfort (possibility to have a seat) and gives the following ranking with respect to five possible alternatives : RER $\succ$ (METRO 1st $\sim$ METRO 2nd) $\succ$ BUS $\succ$ TAXI (see Table 1). As far as the criterion comfort is concerned, a qualitative scale has been used :

0 : no chance of seating
\+ : little chance of seating
++ : great chance of finding a seating place
+++ : seat assured

Means of transportation	Rank	Price (FF)	Time (mn)	Comfort
RER	1	3	10	+
METRO(1)	2	4	20	++
METRO(2)	2	2	20	0
BUS	3	6	40	0
TAXI	4	30	30	+++

TABLE 1 : Ranking and multicriteria evaluations of means of transportation

The first step of UTA* consists of making explicit the utilities of the five alternatives. We have retained the following scales :

$$[g_{i*}, g_i^*] = [30, 16, 2]$$

$$[g_{2*}, g_2^*] = [40, 30, 20, 10]$$

$$[g_{3*}, g_3^*] = [0, +, ++, +++]$$

from which by linear interpolation for the g_1 criterion, we find :

$$u\,[g(RER)] = .07\ u_1(16) + .93\ u_1(2) + u_2(10) + u_3(+)$$

$$u\,[g(METRO1)] = .14\ u_1(16) + .86\ u_1(2) + u_2(20) + u_3(++)$$

$$u\,[g(METRO2)] = u_1(2) + u_2(20) + u_3(0) = u_1(2) + u_2(20)$$

$$u\,[g(BUS)] = .29\ u_1(16) + .71\ u_1(2) + u_2(40) + u_3(0)$$

$$= .29\ u_1(16) + .71\ u_1(2)$$

$$u\,[g(TAXI)] = u_1(30) + u_2(30) + u_3(+++) = u_2(30) + u_3(+++)$$

and according to (10)-(11) we have :

$$u\,[g(RER)] = w_{11} + .93\ w_{12} + w_{21} + w_{22} + w_{23} + w_{31}$$

$$u\,[g(METRO1)] = w_{11} + .86\ w_{12} + w_{21} + w_{22} + w_{31} + w_{32}$$

$$u\,[g(METRO2)] = w_{11} + w_{12} + w_{21} + w_{22}$$

$$u\,[g(BUS)] = w_{11} + .71\ w_{12}$$

$$u\,[g(TAXI)] = w_{21} + w_{31} + w_{32} + w_{33}$$

Finally passing through step 2 of the comparison of alternatives by pair, we arrive at the following linear programme of the new version of UTA (δ equals .05)

w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	Variables σ^+ and σ^-										Sign	Second member
0	.07	0	0	1	0	-1	0	1	-1	-1	1	0	0	0	0	0	0	≥	.05
0	-.14	0	0	0	1	1	0	0	0	1	-1	-1	1	0	0	0	0	=	0
0	.29	1	1	0	0	0	0	0	0	0	0	1	-1	-1	1	0	0	≥	.05
1	.71	-1	0	0	-1	-1	-1	0	0	0	0	0	0	1	-1	-1	1	≥	.05
1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	=	1
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	F	

An optimal solution is :

$$(w_{11} = .5,\ w_{21} = .05,\ w_{23} = .05,\ w_{33} = .4) \text{ with } F^* = \min F = 0,$$

which corresponds to the additive utility

$u_1(30) = 0$	$u_2(40) = 0$	$u_3(0) = 0$
$u_1(16) = .5$	$u_2(30) = .05$	$u_3(+) = 0$
$u_1(2) = .5$	$u_2(20) = .05$	$u_3(++) = 0$
	$u_2(10) = .1$	$u_3(+++) = .4$

and to a perfect numerical restitution of the given weak order.

This solution is not a unique one. Through step 4 (stability analysis) we are looking for multiple optimal solutions or, more generally, for near optimal solutions corresponding to error values between F^* and $F^* + \varepsilon$. We must therefore transform the error objective to a constraint of the type:

$$\sum_{a \in A} \{\sigma^+(a) + \sigma^-(a)\} \leq F^* + \varepsilon \tag{12}$$

In the proposed example, as $F^*=0$ and the linear programme has multiple optimal solutions, we are searching for the more characteristic ones which maximize respectively the quantities (weights) $w_{11} + w_{12}$, $w_{21} + w_{22} + w_{23}$, $w_{31} + w_{32} + w_{33}$. As the total sum of σ^+ and σ^- is zero, we have to solve the following three linear programmes :

	w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	Sign	Second member
	0	.07	0	0	1	0	-1	0	≧	.05
	0	-.14	0	0	0	1	1	0	=	0
	0	.29	1	1	0	0	0	0	≧	.05
	1	.71	-1	0	0	-1	-1	-1	≧	.05
	1	1	1	1	1	1	1	1	=	1
MAX	1	1							←	1st objective
MAX			1	1	1				←	2nd objective
MAX						1	1	1	←	3rd objective

Starting from the optimal solution of the preceeding programme, we obtain three different solutions :

1st solution : (w_{11} = .7625, w_{12} = .175, w_{23} = .0375, w_{31} = .025)

2nd solution : (w_{11} = .05, w_{22} = .05, w_{23} = .9)

3rd solution : (w_{11} = .35625, w_{12} = .175, w_{23} = .0375, w_{31} =.025, w_{33} =.40625)

Let us take the centroid of these three solutions as a unique utility function. Thus, we have :

$u_1(30)=0$ $u_2(40)=0$ $u_3(0)= 0$ u(RER)=.856

$u_1(16)=.39$	$u_2(30)=0$	$u_3(+) = .017$	$u(M1) =.523$
$u_1(2)=.506$	$u_2(20)=.017$	$u_3(++)= .017$	$u(M2) =.523$
	$u_2(10)=.342$	$u_3(+++)=.152$	$u(BUS)=.473$
			$u(TAXI)=.152$

The marginal utilities can be normalized by dividing every utility $u_i(g_i^j)$ by $u_i(g^*_i)$. Then the additive utility is

$$[u(\underline{g}) = .506\ u_1(g_1) + .342\ u_2(g_2) + .152\ u_3(g_3)]$$

with $u_1(g_1)$ and $u_2(g_2)$ as given in figure 2 and :

$$u_3(0) = 0,\ u_3(+) = u_3(++) = .112,\ u_3(+++) = 1$$

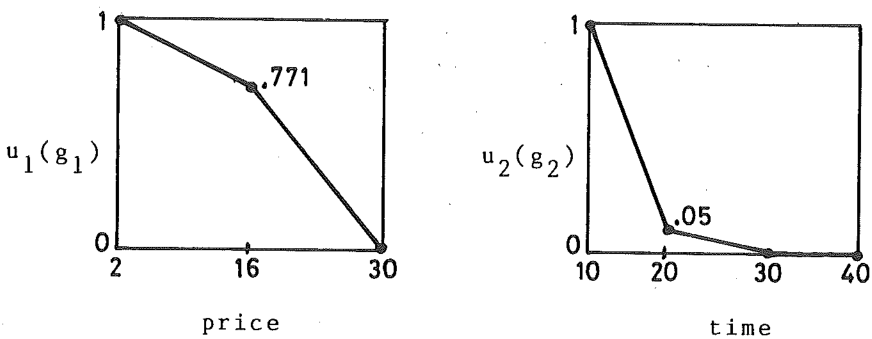

Fig. 2

Marginal utilities of price and time

4. AN EXPERIMENTATION AND CONCLUSION

In order to evaluate the advantages of the UTA* version over the old one, we have considered a set of ten products evaluated on six ordinal criteria (table 2) using a unique scale with four levels, noted : $4 \succsim 3 \succsim 2 \succsim 1$. On these products we have randomly generated around twenty weak orders to analyze by UTA and UTA*. The value of δ parameter has been fixed equal to .05.

Products	Qualitative criteria					
1	1	4	2	3	4	4
2	3	3	3	2	3	2
3	1	4	2	2	4	4
4	2	3	1	3	2	3
5	3	2	2	1	2	2
6	2	4	2	2	4	3
7	4	1	3	2	1	2
8	3	4	2	2	4	3
9	4	1	2	1	2	1
10	4	1	2	1	1	1

TABLE 2 : Experimentation data

For this comparison we have used three indicators : (1) the number of the necessary simplex iteractions for arriving at the optimal utility, (2) the Kendall's τ between the initial weak order and the one produced by the estimated utility and (3) the minimized criterion F (sum of σ) taken here as the indicator of dispersion of the observations. The global results are illustrated in table 3. A net superiority is coming out favorizing the new method, concerning the indicators (1) and (3), which is respectively of the order of 48% and 36%. An amelioration of only 10% is coming out concerning the Kendall's τ. It has been observed that in certain cases the original version assured quite better results for Kendall's τ. The UTA formulation which consists of maximizing this criterion is articulated under the form of a mixed variables linear programme (see [2], p. 162).

Indicator	UTA*	UTA	Balance
Average number of simplex iterations	11.1	21.5	fall 48%
Average Kendall's	.69	.63	increase 10%
Average dispersion index	.34	.53	fall 36%

TABLE 3 : Global results of simulation

We also tested the new version on two large scale examples, stemming from real world studies which had been conducted using the first version of the programme. The first experiment (see [3], p. 198) had 57 alternatives ranked in seven classes and six qualitative criteria. The first regression analysis involved 128 iterations, a. 81 Kendall's τ and 2.38 dispersion indicator, while UTA* gives 129 iterations (+ 1%), Kendall's τ .86 (+ 5%) and dispersion indicator 1.64 (- 31%). The second experiment ([4]) involves 50 alternatives and 12 criteria, only one of them being quantitative. The improvements provided by the new version are summarized as following : number of iterations 135 versus 146 (- 8%), τ.49 versus . 36 (+ 37%) and dispersion 1.11 versus 1.87 (-41%). In the two cases, the impact of the application of UTA* is perceived by a net tightening of the "cloud" of adjusted points around the regression curve. The improvement rate of dispersion index was in all experiments between 30% and 40%.

UTA* has been programmed using the Basic language and works in an interactive way on the ZENITH 100 microcomputer, while UTA exists on an 370/168 IBM system. The UTA approach, especially with its new form, seems to be a very useful tool for interactive modelling of preferences based on the coherence between estimated utilities (human models) and externalized man decisions (see [4]).

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THE DANTZIG WOLFE DECOMPOSITION EXTENDED TO MULTI-CRITERIA LINEAR PROGRAMMING - THEOREMS, PROOFS AND ALGORITHM

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ABSTRACT

The use of a simplex method (either with Zeleny or Philips technique) to find the efficient solutions of a linear multi-objective problem rises increasing time and memory storage difficulties as problems become of large dimension.

In fact, for practical purposes, most authors share the opinion that the process should be controled with the help of other criteria, namely the decision maker's preferences, either in a direct or indirect way.

In any case, every contribution to increase speed and efficiency of the multi-objective simplex is welcome. In this paper, the extension of the Dantzig-Wolfe decomposition to a linear multi-objective problem is presented, along with the theorems and proofs required by an efficient solution search algorithm.

1. INTRODUCTION

The search for the efficient solutions of a linear multi--objective program can be achieved in several ways: by an approximation of the set of the efficient solutions, by iterative solving of single objective problems, or by use of a multi-criteria (multi-objective) Simplex. In this last option, mainly develloped by Zeleny [(1)], an algorithm derived from the single objective Simplex allows one to identify all the efficient solutions, by travelling along the edges of the efficient solution set; however, as the dimension of the constraint matrix increases, time solving problems arise and become critical.

Several authors ([(2)(3)]) for instance have mentioned that these computing problems could be kept at a reasonable level by the use of aproximate methods, or with the previous or interactive inclusion of the decision makers´s preferences in the solving priocess. Without questioning these assertions, this paper presents a technique which is intended to largely reduce computer time and memory requirements when solving a multi-objective Simplex, if a special structure of the constraint matrix can be detected. It is based on the single-objective Dantzig-Wolfe decomposition and on Zeleny multi-objective Simplex, whose fundamentals are shortly described below.

2. DANTZIG-WOLFE DECOMPOSITION

The Dantzig-Wolfe decomposition [(4)] exploits the block--angular structure of the constraint matrix, as follows:

$$\max \quad f = |c_1^T \ \dots \ c_t^T| . x$$

$$\text{subj.} \begin{bmatrix} A_1 & \dots & A_t \\ B_1 & & \\ & \dots & \\ & & B_t \end{bmatrix} . \begin{bmatrix} x_1 \\ .. \\ x_t \end{bmatrix} = \begin{bmatrix} b_0 \\ b_1 \\ .. \\ b_t \end{bmatrix}$$

$$x \geq 0$$

P1: Original Problem

Assuming that the solution set S_k of each subproblem $B_k.x_k=b_k$ is closed and bounded, any element x_k of S_k may be represented by a linear combination such as:

$$X_k = \sum_{j=1}^{V_k} w_k^j . x_k^j \ , \text{ with } \sum_{j=1}^{V_k} w_k^j = 1 \quad (\text{all } k) \quad \text{and } w_k^j \geq 0$$

$$(\text{all } j,k)$$

where x_k^j are the v_k vertices of S_k.

Replacing x in problem P1 leads to the equivalent problem P2, where $u_k^j = c_k^T . x_k^j$ and $p_k^j = A_k^j . x_k^j$

$$\max \qquad f = \sum_{k=1}^{t} \sum_{j=1}^{V_k} w_k^j . u_k^j$$

$$\text{subj.} \quad \sum_{k=1}^{t} \sum_{j=1}^{V_k} w_k^j . p_k^j = b_0$$

$$\sum_{j=1}^{V_k} w_k^j = 1 \quad , \quad k=1..t \text{ and } \quad w_k^j \geq 0 \quad k=1..t, \ j=1..v_k$$

P2: Master Problem

Taking m_o as the number of rows of matrix A_k, and m_k as the number of rows of each matrix B_k, problem P1 has $m_o + \Sigma\, m_k$ constraints, while the master problem P2 has only $m_o + t$ constraints. However, the number of variables of the master is extremely large, as it corresponds to the number of extreme points in S_k for k=1..t. The Dantzig-Wolfe method includes a column generating technique, creating columns as required, which overcomes that difficulty; the process is condensed below.

In the master problem, the reduced cost of $w_k{}^j$ (corresponding to the j extreme point of the k subproblem, $x_k{}^j$) is given by:

$$d_k{}^j = -(c_k{}^T - t_o{}^T . A_k) . x_k{}^j + t_k$$

$$\text{where } t^T = | t_o{}^T \ t_1 \ .. t_t | = u_B{}^T . B^{-1} \text{ and}$$

u_B stands for the vector of the basic variables in the master problem (B^{-1} is the inverse of the basis matrix).

Finding the most negative $d_k{}^j$ is equivalent to solving t subproblems:

$$\begin{array}{ll} \text{max.} & (c_k{}^T - t_o{}^T . A_k) . x_k{}^j \\ \text{subj.} & B_k . x_k = b_k \\ & x_k^j \geq 0 \end{array}$$

P3 - Subproblem k

and determining the minimum of the results.

The algorithm is similar to the usual Simplex method, and the optimum is reached when the $d_k{}^j$ are all non-negative.

The iterated solving of the subproblems P3, with the same constraints but different objective functions (t_0 changes every iteration) can be conducted by post-optimisation.

3. ZELENY MULTI-OBJECTIVE SIMPLEX

The algorithm proposed by Zeleny[1] to determine all the basic efficient solutions of a linear multi-objective program is based on the following theorems, whose proofs can be found in [1] or [2].

T1: If there is a non-basic column j with reduced costs $d_j \leq 0$ (all non-positive with at least one strictly negative), then the present basic solution is dominated.

T2: If the reduced costs of a non-basic column j are $d_j \geq 0$ (all non-negative with at least one strictly positive) then including column j in the basis leads to a dominated solution.

T3: Let j and q be two non-basic columns with reduced costs d_j and d_q, and with minimum replacement rates r_j and r_q, respectively. If $r_j . d_j \leq r_q . d_q$ (all elements less than or equal, with at least one strictly less), then the solution resulting from introducing column q in the basis is dominated by the one resulting from introducing column j in the basis.

Making use of theorems T1 to T3 allows one to select the variables to enter the basis, or to recognize, in some cases, if the present solution is dominated. These criteria, unfortunately, do not cover all cases and an auxiliar problem is then formed:

$$\max \quad Z = \sum_{k=1}^{f} d_k^{+}$$

$$\text{subj.} \quad f_k(x) - d_k^{+} = f_k(x^a) \qquad k=1..f$$

$$d_k^{+} \geq 0 \qquad ,k=1..f$$

$$x \in X$$

P4: Auxiliar Problem

where x^a is the present basic solution, fis the number of objective functions and d_k^{+} are the positive deviations from the present value $f_k(x^a)$ of the k objective function.

If an objective function can be increased without reducing the value of any of the others, then $d_k^{+} > 0$ and $Z^* > 0$, and the present basic solution is dominated. If $Z^*=0$, the present solution is nondominated. We can therefore state the following theorem:

T4: Solve problem P4.
If $Z^* = 0$, the present basic solution x^a is efficient.
If $Z^* > 0$, then x^a is dominated.

4. DECOMPOSITION IN MULTI-OBJECTIVE SIMPLEX

Consider a linear program with a constraint structure similar to problem P1, but with several objective functions. It will be shown that it is possible to form a master problem like problem P2, but multiobjective, whose efficient solutions are also efficient solutions of the original problem. The main diferences to the contents of section 2 lie in the fact that in general the subproblems will also be multi-objective, generating a set of nondominated reduced cost vectors that will be required in the process of choosing the variables of the master problem to enter the basis, according to criteria similar to the multi-objective Zeleny Simplex.

Assume that the set of nondominated solutions of each subproblem k has already been defined:

$$E_k = \{ X_k^1 \ldots x_k^e \ldots x_k^{nk} \}$$

Each of these solutions produces an associated vector of the reduced costs to the master problem:

$$d_k^e = -f_k^e + t_k$$

(The notation convention of section 2. is kept here, taking in account the dimensions increase; f_k^e is the vector of the objective functions values of subproblem k associated with solution x_k^e).

The following problems stand:

T5: Consider a solution w of the master problem, and the corresponding solution x of the original problem. Then, these solutions are both dominated or both non--dominated in their respective problems.

Proof: The construction of the master problem implies that the value of each of its objective functions is the same of the values of the original problem, for the corresponding same solution. The definition of dominated solution completes the proof, if one realizes that every feasible solution of the master problem has a corresponding feasible solution of the original problem, and vice-versa.

T6: If every efficient solution of subproblem k has $d_k^e \geq 0$, then every solution j of subproblem k will have $d_k^j \geq 0$.

Proof: If any dominated solution of k had a reduced cost $(d_k^j)_i < 0$, then:

$$(d_k^j)_i = -(f_k^j)_i + t_{ik} < 0$$

But, for each efficient solution x_k^e we have:

$$(d_k^e)_i = -(f_k^e)_i + t_{ik} \geq 0$$

Therefore

$$-(f_k^j)_i + t_{ik} < -(f_k^e)_i + t_{ik}$$

$$(f_k^j)_i > (f_k^e)_i$$

Then x_k^j cannot be dominated by any efficient solution which means that x_k^j is itself efficient, contradicting the inicial assumption.

From these two theorems, a rejection criterium similar to T2 can be established, deriving directly from theorems T5 and T6:

T7: If all the efficient solutions of subproblem k have $d_k^e \geqq 0$, then the introduction in the master problem of any column (k^j) of subproblem k leads to a dominated solution in the original problem.

The recognition of the optimal solution of each objective function allows one to detect efficient solutions of the multi-objective problem. Thus the importance of the next theorem.

T8: If, for every efficient solution of every subproblem,

$$(d_k^e)_i > 0$$

then the i^{th} objective function of the original is at its unique maximum. If some $(d_k^e)_i = 0$, this maximum is not unique.

Proof: Theorem T6 proof shows that there cannot be in any subproblem k a dominated solution with $(d_k^e)_i < 0$. Therefore, to all the solutions of all the subproblems there is a corresponding positive reduced cost at the i^{th} objective function of the master problem. The optimality criterium of the Simplex method and T5 complete the proof.

Solving the subproblems allows one to eliminate, as candidates to entering the basis of the master problem, the dominated columns of each subproblem. Theorem T7 is a particular case of that feature, which is presented below in a more general formulation.

T9: Introducing, in the basis of the master problem, a column corresponding to a dominated solution of a subproblem generates a dominated solution of the original problem.

Proof: In each subproblem k we have, for every solution x^j_k:
$d^j_k = - f^j_k + t_k$

meaning that the reduced cost vector of the master problem only differs of a constant (for subproblem k) from the objective function vector of subproblem k. Therefore, if a solution of subproblem k is dominated for the reduced cost vector d_k of the master problem. Introducing in the master problem a column corresponding to a dominated solution of a subproblem would then lead to an increase of lesser degree in at least one objective function of the master problem (and of the original problem - T5) than introducing another column corresponding to some other solution dominating, in the subproblem, the solution previously considered. The solution of the original problem would thus be dominated.

The other criteria adopted in Zeleny s algorithm, namely T3, are of straightforward application to the select columns of the subproblems. As for T4, it requires solving an auxiliar problem (P4) related to the rows of reduced costs of the original problem. For practical purposes, the basis has a dimension just equal to the number of objective functions, leading to its direct solving in the original problem.

A simplified algorithm for the application of the decomposition in the multi-objective Simplex is presented at the end of the text.

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REMARK

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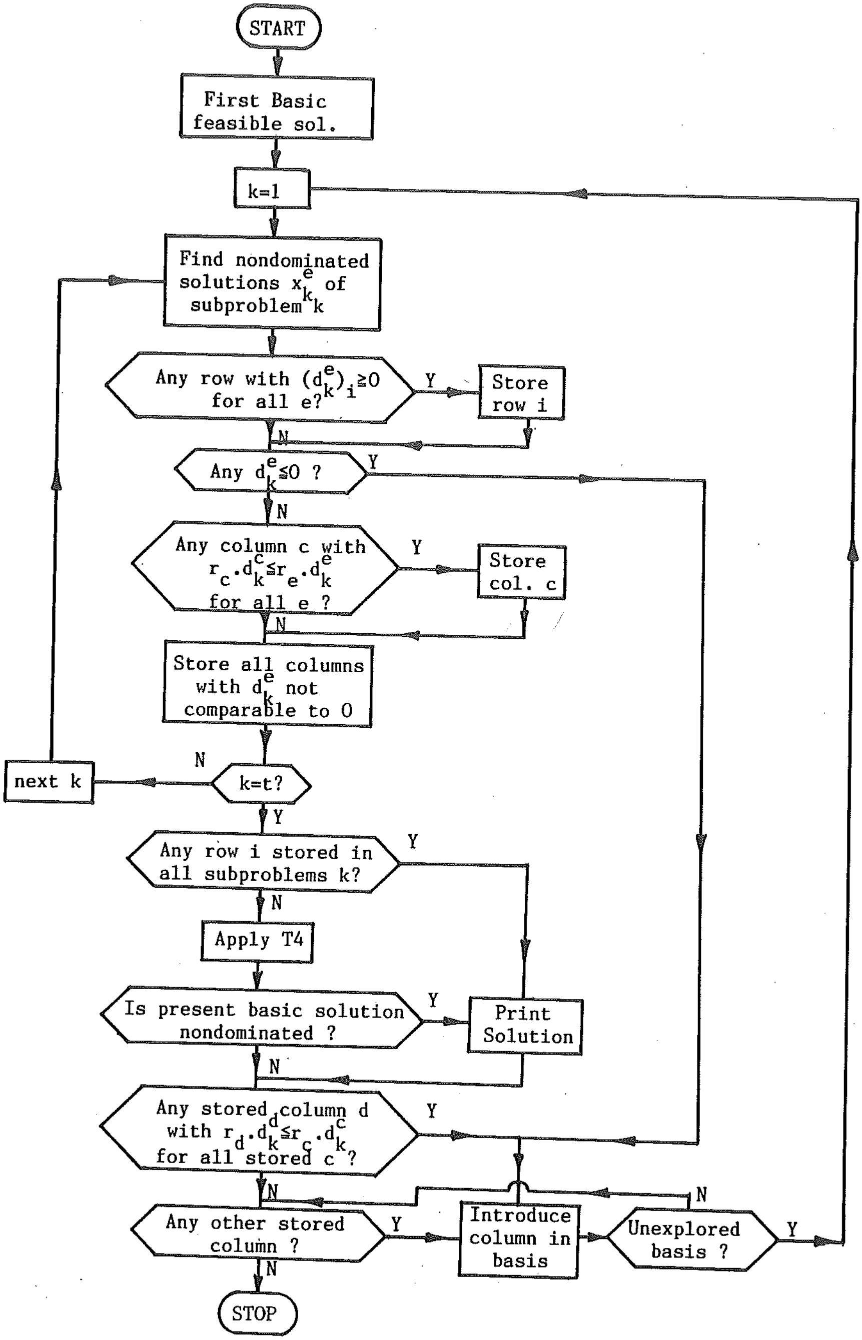

Algorithm for the multi-objective decomposed Simplex

LABOUR STABILITY Vs BUSINESS PROFITABILITY WITHIN AN AGRARIAN REFORM PROGRAMME IN ANDALUSIA (SPAIN): A COMPROMISE PROGRAMMING APPLICATION

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ABSTRACT

In this paper a real problem related to the implementation of the 1984 Agrarian Reform for Andalusia (Spain) is analysed. The problem lies in the degree of conflict between one of the main objectives of the agrarian reform programme: to provide stable employment, and perhaps the main objective of the labourers associated into cooperatives established by the Agrarian Reform: maximisation of business profitability. A compromise between both conflicting objectives in this real case is established by resorting to multiobjective and compromise programming techniques.

Key words: agrarian reform, compromise programming, multiobjective programming, rural employment.

1. THE PROBLEM

One of the main objectives of the 1984 Agrarian Reform Law (ARL) for Andalusia is that it becomes a powerful tool which can mitigate the serious situation of unemployment currently suffered by the rural sector in this autonomous area of Spain. In fact, unemployment in this sector has reached such alarming figures that it has given way to a conflictive social situation with labourer´s occupation of large rural holdings ("latifundios") and a general peasant unrest.

The ARL empowers the Andalusian Institute of Agrarian Reform ("Instituto Andaluz de Reforma Agraria" IARA) to implement, among other measures, the expropriation of rural holdings under certain assumptions of low productivity indexes. These indexes are measures of the level of production, employment, etc. The expropriated holdings will be redistributed among labourers associated into cooperatives.

The IARA will recommend the farm plan to be set up in these cooperatives. In order to choose the optimum cropping pattern the IARA is obviously not interested in the farm plan which maximises the business profitability of the cooperative but in a farm plan that, securing minimum business profitability, maximises the stable employment level.

On the other hand, the main objective for the members of these cooperatives is not the maximisation of the stable employment level but the maximisation of an index of business profitability as the gross margin of the cropping pattern.

Unfortunately, in the case of the crops cultivated in the irrigated lands of Andalusia, these two objectives are in conflict. The objective of providing permanent employment throughout the year can only be achieved for relatively low levels of gross margin. On the contrary, high levels of gross margin are only compatible with low levels of stable employment. In this situation a compromise between these two conflicting objectives must be obtained, establishing the oportunity cost of the stable employment in terms of gross margin.

2. THE STATEMENT OF THE CASE

Let us assume the case of a labourers cooperative in an irrigated arable farm of 100 ha in a certain area of Andalusia under the agrarian reform programme. Table 1 shows the linear programming (LP) matrix for the corresponding farm planning problem. Most of the constraints of the matrix are self-explanatory. However, some of them require further clarification. Thus, constraints (14)-(17) measuring labour seasonality must be explained. These constraints represent the deviations between labour utilisation for each crop in the four periods of the three months considered and the average labour utilisation for each crop. Thus, the first coefficient of row (14) is -59.44 because in the first quarter cotton requires 4.14 hours/ha. while the average labour utilisation for this crop is 63.63 hours/ha/quarter. The deviational variables x_{14} -x_{21} measure the under -and over- achievements with respect to a null deviation in every quarter considered. As is well know, the minimisation of the sum of the deviational variables will imply the minimisation of the mean absolute deviation (see e.g. Hazell 1971 although in quite a different context). Therefore, the minimisation of objective function Z_1 (bottom table 1) implies the minimisation of labour seasonality.

Constraints (18)-(21) guarantee positive cash flows in every quarter. The possibility of transferring the cash surplus in one period to the next has been included in all quarters except in the last one where only 25% of the possible surplus is allowed to be transferred.

The last constraints (22) and (23) secure a private profitability and .a total level of employment to the farm plan of 80.000 pts/ha of gross margin, and 150 hours/ha of employment respectively.

Maximising the objective function Z_2 (see bottom of table 1) the farm plan providing the maximum gross margin (i.e. maximum business profitability) is obtained. This solution provides an optimum level of gross margin 174,116 pts/ha corresponding to a stationality of 253.28 hours/ha. These values and the respective cropping pattern are shown in Table 2 row 6 (point F).

Minimising the objective functions Z_1 (see bottom of Table 1) the farm plan providing the minimum stationality (i.e. maximum labour stability) is obtained. This solution provides an optimum level of stationality of 15.97 hours/ha (practically a complete smoothing of labour utilisation) corresponding to a gross margin of 82,320 pts/ha. These values and the respective cropping pattern can be found in Table 2 row 1 (point A).

These two solutions establish what in multiobjective programming (MOP) literature is called the ideal or utopian point; i.e. the point where all the objectives achieve the optimum value. Thus, in our problem the ideal or utopian point consists in achieving a gross margin of 174,116 pts/ha with a labour stationality of 15.97 hours/ha.

The ideal point in our case is infeasible, as it always happens when the objectives are in conflict, and cannot be chosen. Therefore, it should be chosen as the recommended farm plan to the cooperative the one corresponding to either point F (i.e. a high gross margin with a high seasonality) and point A (i.e. low labour seasonality and low gross margin) or a compromise between these two points.

3. GENERATION OF THE TRADE-OFF CURVE BETWEEN GROSS MARGIN AND LABOUR STABILITY

Points A and F can be considered as the bounds of a certain transformation curve which measures the relationship between gross margin and labour seasonality. To set up that curve could be very useful in order to obtain the trade-offs between the two objectives considered. Establishing the curve is equivalent to generating the set of efficient (non-dominated or Pareto optimal) solutions. The elements of this efficient set are feasible solutions such that there are no other feasible solutions that can achieve the same or better performance for all the objectives and strictly better for at least one objective (e.g. Romero & Rehman 1984, pp 180-81).

The MOP literature offers several approaches to generate or at least to approximate the efficient set: weighthing method, constraint method, multicriterion Simplex, etc. A detailed explanation of these methods can be seen in: Cohon (1978 chap 6), and Goicochea et al. (1982 chap 3).

Among these possible approaches we have chosen the non-inferior set estimation (NISE) method to solve our problem. The NISE method, which was developped by Cohon et al. (1979), permits the exact generation of the efficient set when the number of objectives is two, using a common linear programming code in an iterative way.

Applying the NISE method to the bicreteria linear programming problem presented in Table 1, the trade-off curve or efficient set between gross margin an labour seasonality of figure 1 is obtained. The coordinates of these extreme points and the values of the decision variables (cropping patterns) are shown in Table 2.

The actual values of the trade-offs between gross margin and labour seasonality can be viewed as the slopes of the straight lines connecting the extreme efficient points in Fig. 1. Thus, the slope of the segment AB indicates that in this part of the trade-off curve for each hour/ha of increase in labour seasonality gross margin increases in 1590 pts/ha.

The optimum farm plan should be chosen by the IARA from the trade-off curve or set of efficient solutions. But, which efficient farm plan will be chosen by the IARA? The answer will depend on the preferences that the IARA attaches to each objective; i.e. it will depend on the subjective values of trade-offs between gross margin and labour seasonality. For instance, if the IARA chooses the farm plan given by point B instead of the one given by point A it will mean that for the IARA a reduction of 21.49 hours/ha of labour seasonality does not compensate a decrease of 34,110 pts/ha of gross margin.

4. THE COMPROMISE SET BETWEEN BUSINESS PROFITABILITY AND LABOUR STABILITY

Once the trade-ff curve or efficient set has been defined the following step in our analysis is to established the optimum efficient point or at least to reduce the size of the efficient set. These purposes can be achieved resorting to a compromise programming (CP) approach. This approach, proposed by Zeleny (1973, 1974 and 1976) helps to choose the optimum element from a set of efficient solutions. Its basic idea is to define the optimum solution as the efficient solution that is closest to the ideal point (axiom of choice, Zeleny 1976 p 171). Depending on the particular measure of distance used, a compromise set (subset of the efficient set) can be established.

In our case we need to calculate distances from every point of the trade-off curve to the ideal point. With this purpose in mind the degree of closeness d_j between jth objective and its ideal is given by:

$$d_j = Z^*_j - Z_j\ (\underline{x})$$

when jth objective is maximised, or as

$$d_j = Z_j\ (\underline{x}) - Z^*_j$$

when the jth objective is minimised, Z^*_j being the ideal value. When the units used to measure various objectives are different (pesetas and hour in the case considered), relative deviations rather than absolute ones must be used (Zeleny 1973, p 299). Thus the degree of closeness is given by:

$$d_j = \frac{Z^*_j - Z_j(\underline{x})}{Z^*_j - Z_{*j}}$$

or

$$d_j = \frac{Z_j(\underline{x}) - Z^*_j}{Z_{*j} - Z_j}$$

where Z_{*j} is the anti-ideal for jth objective; i.e. the value of jth objective when the conflicting objective is optimised (for our case 229.90 hours/ha for labour seasonality and 82,320 pts/ha for gross margin).

In order to obtain the distances between each solution and the ideal point CP introduces the following family of distance functions:

$$L_P(\partial, k) = \left[\left(\sum_{j=1}^{k} \partial_j \; d_j\right)^P\right]^{1/P}$$

where ∂_j weights the importance of the discrepance between the jth objective and its ideal value.

For the metric L_1, i.e. for P=1, the best-compromise solution is found by solving the following linear programming (LP) problem (see e.g. Cohon 1978 p. 1985):

$$\text{Min } L_1 = \frac{\partial_1 \; [Z_1(\underline{x}) - 15.97]}{229.90 - 15.97} + \frac{\partial_2 \; [174.116 - Z_2(\underline{x})]}{174.116 - 82.320}$$

which is tantamoumt to:

$$\text{Min } L_1 = \frac{1}{213.93} Z_1(\underline{x}) - \frac{2}{91.796} Z_2(\underline{x}) \qquad (1)$$

s.t. $\underline{x} \in \underline{F}$ (technical constraints from Table 1).

The optimum solution of problem (1) for $\partial_1 = \partial_2$; i.e. assuming that the two objectives are equally important, is given by point C. Therefore, point C is the best-compromise solution and this means that C is the efficient point closest to the ideal point when the metric L_1 is used.

For the metric L_∞, i.e. for $P = \infty$, the maximum of the individual deviations is minimised. That is when $P = \infty$ only the largest deviation counts. For this metric, the best-compromise solution is found by solving the following LP problem (see e.g. Cohon 1978, pp. 185-187):

$$\text{Min } L_\infty = d_\infty$$

$$\text{s.t.} \quad \frac{\partial_1 \left[Z_1(\underline{x}) - 15.97\right]}{229.90 - 15.97} \leq d_\infty$$

$$\frac{\partial_2 \left[174.116 - Z_2(\underline{x})\right]}{174.116 - 82.320} \leq d_\infty \qquad (2)$$

$\underline{x} \in \underline{F}$ (technical constraints from Table 1)

where d_∞ is the largest deviation. The optimum solution of problem (2) assuming again $\partial_1 = \partial_2$, is given by point S. Figure 1 and the last row of Table 2 shows the values of Z_1, Z_2 and the farm plan corresponding to point S.

Yu (1973) has demonstrated that metrics L_1 and L_∞ define a subset of the efficient set, which Zeleny (1974, p. 488) calls the compromise set. The other best solution (for $1 \leqq P \leqq \infty$) fall between the solutions corresponding to metrics L_1 and L_∞.

Segment $\overline{CS}$ represents the compromise set. The optimum solution will be chosen by the IARA from the points belonging to that segment. Obviously, if the weights ∂_1 and ∂_2 attached to the discrepancies between each objective and its ideal value are different with respect to the values considered in this case ($\partial_1 = \partial_2$), the structure of the compromise set can be modified. Performing a sensitivity analysis with the ∂_j weights can furnish the decision maker with worthwhile data related to the range where the compromise set can be defined.

The last column of Table 2 shows the level of employment corresponding to the six efficient extreme points and to point S. These figures have been included for two reasons. First, to evaluate the opportunity cost of the different policies in terms of gross margins. Second, to provide additional information in order to choose one point belonging to the compromise set. Thus, in our case point S perhaps should be chosen because its level of employment is 60.8 hours/ha. higher than the employment of point C.

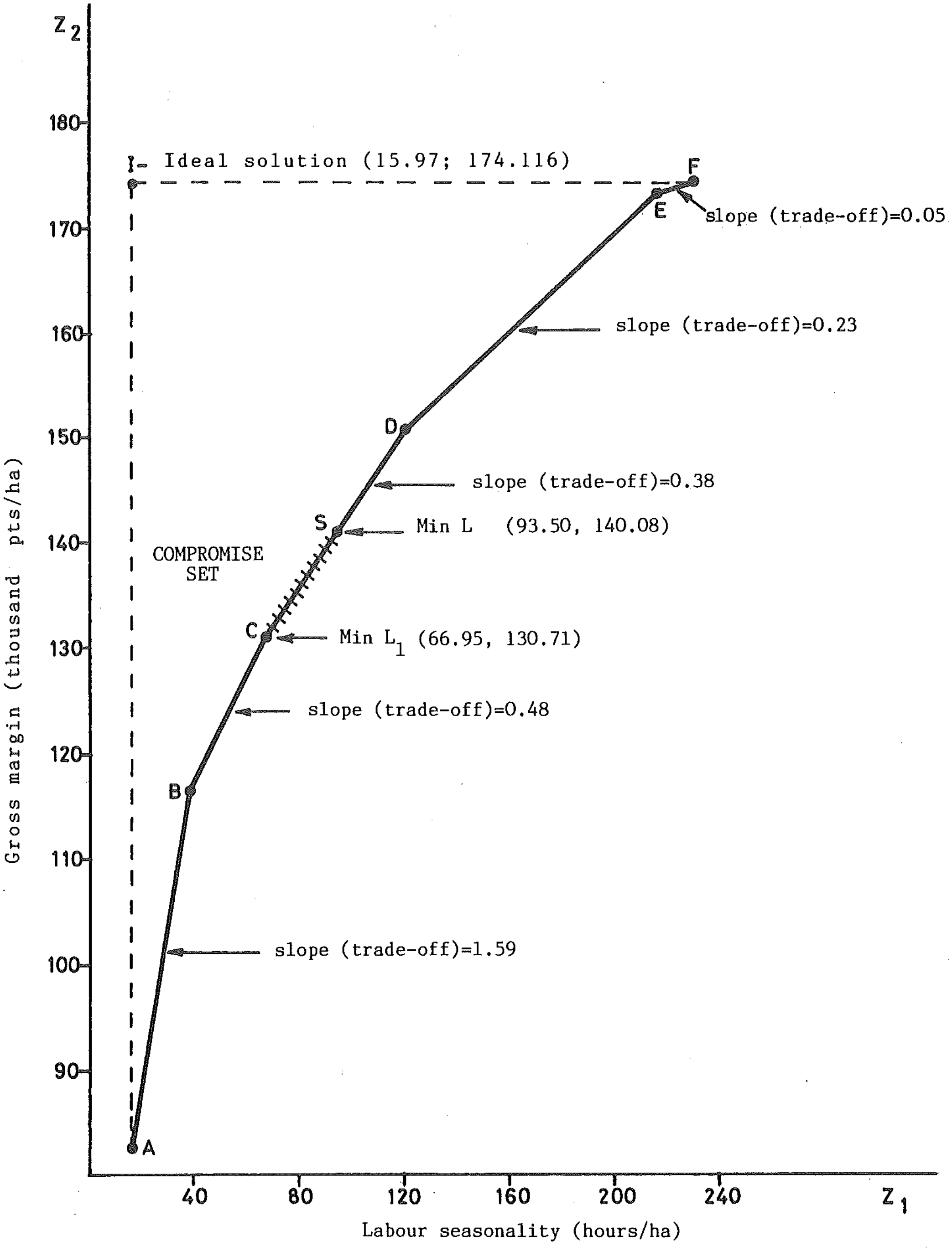
TABLE I

THE BICRITERIA LINEAR PROGRAMMING MODEL THE CASE OF AN IRRIGATED ARABLE FARM IN SEVILLE

REAL ACTIVITIES (HA)													LABOUR SEASONALITY DEVIATIONAL VARIABLES (HOURS)								WORKING CAPITAL TRANSFER ACTIVITIES (THOUSAND PTS)							
Cotton	Wheat	Corn	Soybean	Potatoes	Sugar beet	Soybean Wheat	Potatoes System	Potatoes Corn	Sorghum	Lettuces	Lettuces Corn	Peach trees																
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}				
1																									$\leqslant$	30	Cotton Maximum (ha)	(1)
	1					1																			$\leqslant$	25	Wheat Maximum (ha)	(2)
		1						1			1														$\leqslant$	25	Corn Maximum (ha)	(3)
			1			1	1																		$\leqslant$	25	Soybean Maximum (ha)	(4)
				1			1	1																	$\leqslant$	15	Potatoes Maximum (ha)	(5)
					1																				$\leqslant$	20	Sugar Beet Maximum (ha)	(6)
										1	1														$\leqslant$	10	Lettuces Maximum (ha)	(7)
												1													$\leqslant$	15	Peach Trees Maximum (ha)	(8)
10	2	12		9.5	5	2	9.5	21.5	6	46	58	4.5													$\leqslant$	750	Tractor hours - Period I	(9)
5	2	6	3	10		5	13	16	7		6	4.5													$\leqslant$	750	Tractor hours - Period II	(10)
3.5		3	8		5.25	8	8	3	17	45	48	4													$\leqslant$	750	Tractor hours - Period III	(11)
2	9		2.5	17	15	11.5	19.5	17		52	52	6													$\leqslant$	750	Tractor hours - Period IV	(12)
1	1	1	1	1	1	1	1	1	1	1	1	1													=	100	Crop land (ha)	(13)
-59.49	- 4.57	-25.32	- 8.06	-23.33	31.24	-12.64	-31.88	-48.65	-24.35	150.10	124.78	-35.94	1	-1											=	0	Labour seasonality (hours) - Period I	(14)
- 5.40	6.73	55.91	- 7.03	139.24	-21.53	- 0.30	132.21	195.15	53.76	-197.50	-141.59	571.68			1	-1									=	0	Labour seasonality (hours) - Period II	(15)
43.59	1.08	- 8.21	20.80	-87.57	90.13	-21.88	-66.77	-95.78	- 7.89	-71.1	-79.31	-331.71					1	-1							=	0	Labour seasonality (hours) - Period III	(16)
21.29	- 3.24	-22.42	- 5.71	-28.35	-99.78	- 9.25	-34.06	-50.77	-16.65	118.5	96.79	-235.05							1	-1					=	0	Labour seasonality (hours) - Period IV	(17)
- 2.953	- 3.207	- 2.822		-40.612	-68.085	- 3.207	-40.612	-43.434	- 2.764	88.863	86.040	-87.457									-1			0.25	$\geqslant$	0	Working capital (thousand pts/ha) - - Period I	(18)
-41.450	79.950	-72.653	- 2.078	405.823	-37.961	77.872	403.745	509.210	-71.153		-72.653	712.797									1	-1			$\geqslant$	0	Working capital (thousand pts/ha) - - Period II	(19)
-76.342	-12.301	158.506	61.861	- 3.631	231.992	-70.440	-61.770	-21.165	142.828	-49.737	108.379	-10.761										1	-1		$\geqslant$	0	Working capital (thousand pts/ha) - - Period III	(20)
252.043	- 3.270	- 5.320	- 4.748	-37.715	- 3.780	111.982	77.537	133.005	- 5.210	145.656	140.336	-30.827											1	-1	$\geqslant$	0	Working capital (thousand pts/ha) - - Period IV	(21)
254.50	26.25	114.40	32.25	375.50	434	58.50	407.75	489.90	110	790	904.40	1356													$\geqslant$	1500	Employment Hours	(22)
107.128	53.780	79.385	50.815	315.973	117.551	104.595	365.918	384.424	55.298	196.200	275.511	332.877														8000	Gross margin (thousand pts)	(23)
													1	1	1	1	1	1	1	1						Z_1	Minimise Labour Seasonality	(Z_1)
107.128	53.780	79.385	50.815	315.973	117.551	104.595	365.918	384.424	55.298	196.200	275.511	332.877														Z_2	Maximise Gross Margin (thousand pts)	(Z_2)

TABLE 2. Feasible Efficient extreme Points and Cropping Patterns

Extreme points	Objective Functions: Z_1 (labour seasonality) hours/ha	Objective Functions: Z_2 (Gross margin) thousand pts/ha	DECISION VARIABLES: x_1 cotton -ha-	x_2 Wheat -ha-	x_3 Corn -ha-	x_4 Soybean -ha-	x_5 Potatoes -ha-	x_6 Sugar beet -ha-	x_7 Soybean + wheat -ha-	x_8 Potatoes + Soybean -ha-	x_9 Potatoes + corn -ha-	x_{10} Sorghum -ha-	x_{11} -ha-	x_{12} Lettuces + corn -ha-	x_{13} Peach trees -ha-	EMPLOYMENT hours/ha
A	15.97	83.320	16.43	25	12.45	25	2.10	4.05	–	–	–	8.56	6.41	–	–	156.18
B	37.46	116.430	27.72	11.57	–	25	15	1.92	–	–	–	13.66	4.64	–	0.49	204.63
L_1 – C	66.95	130.709	30	–	11.49	–	15	8.37	25	–	–	9.72	0.42	–	–	210.78
D	121.23	151.088	30	3.63	13.66	10	–	20	–	15	–	1.08	–	–	6.63	335.23
E	216.71	173.424	30	–	12.08	8.91	–	19.01	–	15	–	–	–	–	15	440.13
F	229.90	174.116	30	–	8.81	–	–	12.83	10	15	–	8.36	–	–	15	421.73
L – S	93.50	140.08	30	–	15.80	6.56	11.11	14.78	14.55	3.89	–	–	–	–	3.31	271.58



Z_2
Gross margin (thousand pts/ha)
180
170
160
150
140
130
120
110
100
90
I Ideal solution (15.97; 174.116)
F
E
slope (trade-off)=0.05
slope (trade-off)=0.23
D
slope (trade-off)=0.38
S
Min L (93.50, 140.08)
COMPROMISE SET
C
Min L_1 (66.95, 130.71)
slope (trade-off)=0.48
B
slope (trade-off)=1.59
A
40
80
120
160
200
240
Z_1
Labour seasonality (hours/ha)

FIGURE 1

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STRANGE: AN INTERACTIVE METHOD FOR MULTI-OBJECTIVE LINEAR PROGRAMMING UNDER UNCERTAINTY

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ABSTRACT

In the field of investment planning within a time horizon, problems typically involve multiple decision objectives, and basic data are uncertain. In a large number of cases, these decision problems can be written as linear programming systems in which time dependent uncertainties affect the coefficients of objectives and the RHS of the constraints. Given the possibility of defining plausible scenarios on basic data, discrete sets of such coefficients are given, each with its subjective probability of occurrence. The corresponding structure is then characteristic for Multi-Objective Stochastic Linear programming (MOSLP).

In the paper, an interactive procedure is described to obtain a best compromise for such a MOSLP problem. This algorithm, called STRANGE, extends the STEM method to take the random aspects into account. It involves in particular, the concepts of stochastic programming with recourse. In its interactive steps, the efficiency projection techniques are used to provide the decision-maker with detailed graphical information on efficient solution families.

As an illustration of the successive steps, a didatic example is solved in some detail.

1. INTRODUCTION

Many practical problems such as investment planning on a long time horizon, can be formulated as multi-objective linear programming problems, having some coefficients affected by uncertainty. An example has been described in detail in [7].

It concerns small rural communities, without industrial activities, far removed from the grid in a Third World Country with important solar resources. Roughly speaking, the problem consists of comparing the merits of electrosolar generators with those of traditional Diesel systems and of defining a strategy for the planning of the electrical power system over a time horizon of twenty years. The linear model-lization represents the dynamic implementation of these two types of equipment, which being subjected to certain techno-logical constraints, must be adequate to satisfy both the energy and power demand of the community.

Three objectives are considered: the total production cost, the outside expenses and the safety of energy supply. It is obvious that some coefficients in this problem are uncertain:

- First of all, from one scenario to another, there are variations in both the Diesel fuel price and in the cost of solar technology.

- Similarly, different assumptions are made with regards to the fuel supply conditions and the statistics concerning the number of days of insolation available.

For each scenario, probabilities of occurrence are defined by the experts. It must be made clear that these probabilities are subjective. They generally at least account for mean and extreme situations, and it is usually difficult to define the occurrence of a given scenario precisely.

Hence, at the end of the study, it is necessary to carry out a sensitivity analysis simulating events with a range of subjective probabilities.

Another application of this type will be described in a forthcoming paper [6].

Such problems result in a particular structure of multi-objective and stochastic linear programming (MOSLP), in which discrete random coefficients are present in the objective functions and in the RHS of some constraints; yet, the coefficients of the LHS of the constraints are related to the technology of the problem and are generally deterministic.

So, we obtain (see [11]), a MOSLP problem formulated as follows:

$$\begin{cases} \text{"min" } z^k = c^k \cdot X & k = 1,\dots,K \\ X \in D(X) = \{X \mid T\,X \leq d,\ X \geq 0\} \end{cases} \qquad (1)$$

where c^k and d are "discrete random variables"; more precisely:

- The k-th linear objective function depends on S_k different scenarios, each of them being affected by a subjective probability or level of plausibility; let c^{ks_k} ($s_k = 1,\ldots,S_k$) be the possible values of c^k and $p^{(k)}_{s_k}$, the subjective probability of scenario s_k, with

$$\sum_{s_k=i}^{S_k} p^{(k)}_{s_k} = 1.$$

- Some elements of vector d are uncertain; let d^r ($r = 1,\ldots,R$) be the possible outcomes of vector d and q_r, the corresponding subjective probability, with

$$\sum_{r=1}^{R} q_r = 1.$$

In the literature, very few papers exist concerning MOSLP. Recently, Stancu-Minasian [10] has presented a survey of this subject. The most commonly used approach seems to be the "Protrade method" of Goicoecha [4]. This method concerns the problem:

$$\left[\begin{array}{ll} \text{"min" } z^k(\omega) = ck(\omega) \,.\, X & k = 1,\ldots K \\ \quad X \in \bar{D}(X) = \{X \mid g(X) \leqq 0,\ X \geqq 0\} & \end{array}\right.$$

where vector g(X) is composed of differentiable and convex functions.

The great advantages of this method are its being interactive and its treatment of a very general problem, with non-linear constraints and general distribution for the random coefficients of the linear objectives. However, in our opinion, it presents several disadvantages; especially:

- a utility function is introduced, the practical construction of which is often a difficult problem; moreover, the use of such a function is based on certain assumptions which are often unsatisfied in practice and thus limit its area of application;

- at each step, the information is reduced to the mean value of the objectives, hence not taking the dispersion enough into account. This can be insufficient for the decision maker to correctly appreciate a compromise.

Other technical characteristics of this make it difficultly to apply to problems such as those described in [6] and [7]. Hence, in this paper, we present STRANGE, a new interactive method for the treatment of the problem (1).

In section 2, we shall transform the problem into an equivalent deterministic multi-objective LP problem and describe how to obtain a first compromise; Section 3 is devoted to the interactive phases and special attention will be given to the collection of precise information which can help the decision-maker to take a more calculated choice. Some conclusions are presented in Section 4.

Each of these sections will be illustrated by a didactic example defined in Section 2.

Remark:

Another way of treating multiple criteria LP problems with uncertain coefficients is to introduce fuzzy numbers to characterize the uncertainty. This approach results in the formulation of a multi-oibjective fuzzy linear programming (MOFLP) system. Several papers have been published on this subject; for instance, Carlsson [2] uses fuzzy programming to solve ill-structured problems, and recently, Slovinsky |9| has carried out a survey of different methods to fuzzy linear programming. Further, in this paper, this author presents a case study involving the development planning for a water supply system, for which he proposes a new MOFLP algorithm.

2. THE FIRST COMPROMISE PHASE

Example: Let us consider the following didactic example, in a two-dimensional space, with a similar structure to that of problem (1).

First objective z^1

$$z^1 = 15 + c^1.(x_1,x_2)'$$

Three scenarios affect this objective ($S_1 = 3$):

$$\begin{cases} s_1 = 1 : c^{11}=(2,-4) & \text{with probability } p_1^{(1)} = \frac{1}{4} \\ s_2 = 2 : c^{12}=(-\frac{5}{2}, \frac{-7}{2}) & \text{with probability } p_2^{(1)} = \frac{1}{2} \\ s_3 = 3 : c^{13}=(3,-3) & \text{with probability } p_3^{(1)} = \frac{1}{4} \end{cases}$$

Second objective z^2

$$z^2 = \frac{109}{16} + c^2.(x_1,x_2)'$$

Two scenarios affect this objective ($S_2 = 2$):

$$\begin{cases} s_2 = 1 : c^{21} = (-1, \frac{3}{4}) & \text{with probability } p_1^{(2)} = \frac{1}{4} \\ s_2 = 2 : c^{22} = (-\frac{3}{4}, 1) & \text{with probability } p_2^{(2)} = \frac{3}{4} \end{cases}$$

Constraints:

The constraints are defined by

- The matrix $T = \begin{pmatrix} -7 & 12 \\ 1 & -1 \\ 1 & 3 \end{pmatrix}$

- The discrete random variable d, with four possible outcomes

$r = 1 : d^1 = (39,5,18)'$ with probability $q_1 = \frac{4}{10}$

$r = 2 : d^2 = (39,5,15)'$ with probability $q_2 = \frac{3}{10}$

$r = 3 : d^3 = (39,5,12)'$ with probability $q_3 = \frac{2}{10}$

$r = 4 : d^4 = (39,5,9)'$ with probability $q_4 = \frac{1}{10}$

Only the RHS of the third constraint is uncertain. The set D(X) is thus given as in Fig. 1, where the interrupted lines represent the four possible outcomes due to the randomized constraint.

2.1. An equivalent deterministic multi-objective LP problem

First, each situation (k, s_k) is defined as a criterion, to take into account the different scenarios affecting the K objectives, i.e.

$$z^{ks_k}(X) = c^{ks_k}.X \qquad k = 1,\ldots,K \; ; \; s_k = 1,\ldots,s_k$$

These criteria will be seen as "new objectives".

Then the difficulty related to the random RHS d is treated. Using the idea of "stochastic programming with recourse" [5], slack variables are introduced measuring the violation of the constraints, corresponding to the realization d^r of d:

$$T\ X + V^r - W^r = d^r \qquad r = 1,\ldots,R$$

resulting in

$$D_1(Z) = \{Z = (X, V^r, W^r,\ r=1,\ldots,R) \mid T\ X + V^r - W^r = d^r,\ Z \geq 0\}$$

Then, as in [8], a supplementary criterion is created to penalize the violation of the constraints:

$$\min \sum_{r=1}^{R} q_r(b^r.W^r) \tag{2}$$

where b^r is a vector of linear penalties.

This criterion is not affected by the different scenarios and to unify the notations, it has been defined as:

$$z^{K+1,s_{K+1}}(Z) \qquad \text{with } S_{K+1} = 1$$

So that, the following deterministic multi-objective LP problem is obtained:

$$\left[\begin{array}{l} \text{"min" } z^{k.s_k}(Z) \\ \\ \qquad Z \in D_1(Z) \end{array}\right. \qquad k = 1,\ldots, K+1\ ;\ s_k = 1,\ldots,S_k \tag{3}$$

with $\sum_{k=1}^{K+1} S_k$ criteria.

<u>Example</u>:

The structure (3) corresponds to the following multi-objective LP problem, with the following constraints:

$$\begin{cases} -7\,x_1 + 12\,x_2 \leq 39 \\ x_1 - x_2 \leq 5 \\ x_1 + 3\,x_2 + v^1 = 18 \\ x_1 + 3\,x_2 + v^2 - w^2 = 15 \\ x_1 + 3\,x_2 + v^3 - w^3 = 12 \\ x_1 + 3\,x_2 + v^4 - w^4 = 9 \end{cases} \qquad x_1, x_2 \geqq 0 \;;\; v^i \geqq 0, w^i \geqq 0$$

and the six following criteria to minimize:

$$z^{11} = 15 + 2\,x_1 - 4\,x_2$$

$$z^{12} = 15 + \frac{5}{2}\,x_1 - \frac{7}{2}\,x_2$$

$$z^{13} = 15 + 3\,x_1 - 3\,x_2$$

$$z^{21} = \frac{109}{16} - x_1 + \frac{3}{4}\,x_2$$

$$z^{22} = \frac{109}{16} - \frac{3}{4}\,x_1 + x_2$$

$$z^{31} = 1 + \frac{3}{10} w^2 + \frac{2}{10} w^3 + \frac{1}{10} w^4$$

Remarks

(i) w^1 is set to zero as it is necessary to at least verify $x_1 + 3x_2 \leq 18$.

(ii) The linear penalties are taken as being equal to one.

(iii) A constant is introduced in z^{31} to always have strictly positive values for the criteria.

2.2. The pay-off table

For each of the $\left(\sum_{k=1}^{K+1} S_k\right)$ criteria $z^{ks_k}(Z)$, an optimal solution $\tilde{Z}^{ks_k}$ of the corresponding single criterion LP problem is determined; this provides an ideal point in the criterion space:

$$M^{ks_k} = z^{ks_k}(\tilde{Z}^{ks_k}) \qquad (4)$$

- If $\tilde{Z}^{1t_1}$ is a unique solution of the problem $(1,t_1)$, the value is introduced:

$$\bar{Z}^{(1t_1)(ks_k)} = z^{ks_k}(\tilde{Z}^{1t_1}) \qquad (5)$$

- Otherwise, as clearly explained in [3], the optimal solution of one of the single criterion problem being not unique, the values defined by (5) are underdetermined. Note that this often happens in practice, since some variables are irrelevant with respect to a particular objective (see, for instance, the objective describing the raw material supply in [6]). Moreover, it is always the case for the criterion K+1 defined by (2).

Then, the problem $(1,t_1)$ is degenerated, the updated technique proposed by Despontin [3] is applied to avoid ambiguity in the resulting pay-off table. The single criterion problem is solved:

$$\left[\begin{array}{l} \min z^{ks_k}(Z) \\ \\ \quad Z \in D_1(Z) \\ \quad z^{1t_1}(Z) = M^{1t_1} \end{array}\right. \qquad (6)$$

and $\bar{z}^{(1t_1)(ks_k)}$ is defined as the optimal value of (6).

The pay-off table is formed of all the values $\bar{z}^{(1t_1)(ks_k)}$ given by (5) and (6), with the co-ordinates of the ideal point on the diagonal of the table.

<u>Example</u>:

Each of the criteria z^{11}, z^{12}, z^{13}, z^{21}, z^{22} has a unique optimal solution, respectively:

$$\tilde{X}^{11} = (\ 3\ ,\ 5\ )'$$

$$\tilde{X}^{12} = (\ 0\ ,\ \frac{13}{4}\ )'$$

$$\tilde{X}^{13} = (\ 0\ ,\ \frac{13}{4}\ )'$$

$$\tilde{X}^{21} = (\ \frac{33}{4}\ ,\ \frac{13}{4}\ )'$$

$$\tilde{X}^{22} = (\ 5\ ,\ 0\ )'$$

so that the corresponding columns of the pay-off table are immediately obtained by (5).

Clearly, it is not the case for criteria z^{31}: we have $M^{31} = 1$ and the corresponding column of the pay-off table is determined with (6).

The pay-off table is then as follows:

	X^{11}	X^{12}	X^{13}	X^{21}	X^{22}	$\tilde{X}^{31}$(*)
z^{11}	1	2	2	$\frac{37}{2}$	25	3
z^{12}	5	$\frac{29}{8}$	$\frac{29}{8}$	$\frac{97}{4}$	$\frac{55}{2}$	$\frac{9}{2}$
z^{13}	9	$\frac{21}{4}$	$\frac{21}{4}$	30	30	6
z^{21}	$\frac{121}{16}$	$\frac{37}{4}$	$\frac{37}{4}$	1	$\frac{29}{16}$	$\frac{25}{16}$
z^{22}	$\frac{153}{16}$	$\frac{161}{16}$	$\frac{161}{16}$	$\frac{31}{8}$	$\frac{49}{16}$	$\frac{49}{16}$
z^{31}	4	$\frac{41}{40}$	$\frac{43}{40}$	4	1	1

(*) This column correspond to different optimal solutions X^{31}; these solutions are in fact, for each criterion the optimal solution submitted to the constraints

$$\begin{cases} -7\,x_1 + 12\,x_2 \leq 39 \\ x_1 - x_2 \leq 5 \\ x_1 + 3\,x_2 \leq 9 \end{cases} \qquad x_1 \geq 0,\ x_2 \leq 0$$

2.3. The first compromise

To determine the first compromise, a minimax optimization is applied like in the STEM method [1]; however, in our case, as all criteria (k,s_k), $s_k = 1,\dots,S_k$, correspond to different scenarios of a same objective, we have to minimize the maximum <u>mean</u> distance between the ideal point and this first compromise, i.e.

$$\left[\begin{array}{l} \min \delta \\ \sum_{s_k=1}^{S_k} P_{sk}^{(k)} \left(c^{ks_k} \cdot Z - M^{ks_k}\right) \pi^{ks_k} \leq \delta \quad k=1,\dots,K+1 \\ Z \in D_1(Z) \end{array}\right. \tag{7}$$

where the weights are determined in a similar way as in [1]

$$\pi^{ks_k} = \frac{\alpha^{ks_k}}{\sum_{k=1}^{K+1} \sum_{s_k=1}^{S_k} \alpha^{ks_k}}$$

where

$$\alpha^{ks_k} = \frac{m^{ks_k} - M^{ks_k}}{m^{ks_k}} \cdot \frac{1}{\| c^{ks_k} \|}$$

with

$$m^{ks_k} = \max_{(1t_1)} \bar{z}^{(1t_1)(ks_k)} \tag{8}$$

However, in case the optimal solution (7) is not unique, the maximax solution obtained may not be efficient; to ensure the efficiency of the first compromise, the Despontin modification [3] might be used, giving the problem P_1:

$$P_1 : \begin{cases} \min M \cdot \delta - \sum_{k=1}^{K+1} \epsilon_k \\ \sum_{s_k=1}^{S_k} p_{s_k}^{(k)} \left(c^{ks_k} \cdot Z - M^{ks_k}\right) \cdot \pi^{ks_k} \leq \delta - \epsilon_k \quad k=1,\ldots,K+1 \\ Z \in D_1(Z) \qquad (9) \\ \epsilon_k \geq 0 \quad k=1,\ldots,K+1 \end{cases}$$

where M stands for a very large number, like in the method of artificial variables of linear programming.

Let us call $\tilde{Z}_1$, the first compromise given by the optimal solution of P_1.

<u>Example</u>:

The following weights are obtained (taking into account terms in the objective functions):

$\pi^{11} = 0.055$; $\pi^{12} = 0.052$; $\pi^{13} = 0.050$;

$\pi^{21} = 0.184$; $\pi^{22} = 0.143$;

$\pi^{31} = 0.516$

so that the first compromise Z_1 given by (9) is :

$x_1 = 4.54 \quad ; \quad x_2 = 3.40$

$v^2 = 0.27$

$w^2 = 0 \quad ; \quad w^3 = 2.73 \quad ; \quad w^4 = 5.73$

This result is shown in Fig. 2.

3. THE INTERACTIVE PHASE

For each compromise $\tilde{Z}_m$, the decision-maker receives three pieces of information:

(1) The value of objective k in the situation of scenario s_k

$$\bar{z}^{ks_k} = z^{ks_k}(\tilde{Z}_m)$$

which is given, regarding the interval of variability of this criterion:

$$[M^{ks_k}, m^{ks_k}]$$

This first set of information has the advantage of not reducing the complexity of the problem, hence leaving the decision-maker with a complete view on the consequences of a compromise.

(2) However, it can be useful to also provide the mean value of each objective k

$$\bar{\bar{z}}^k = \bar{z}^k(\tilde{Z}_m)$$

with

$$\bar{z}^k(Z) = \sum_{s_k=1}^{S_k} p_{s_k}^{(k)} z^{ks_k}(Z)$$

(3) The decision-maker is also very interested by the variation of each objective. To give him a simple idea of it, the confidence level of the compromise is determined, i.e.

$$1 - \alpha_m^k = P\,(c^k \,.\, \tilde{Z}_m \leq \bar{z}_m^k) = \sum_{\substack{s_k \\ \bar{z}^{ks_k} \leq \bar{z}_m^k}} p_{s_k}^{(k)}$$

If the decision-maker is satisfied with the compromise, the procedure stops. Otherwise, one asks him to indicate:

- a criterion $(ks_k)^*$ to be relaxed;

- and upper limit $\Lambda^{(ks_k)^*}$ of the increase of $z_m^{(ks_k)^*}$.

<u>Remark</u>:

The first indication is essential as it gives a direction in which the compromise will be improved; in practice, if the decision-maker is generally able to indicate the most favourable candidate for such relaxation,

he has often some difficulty to expresss $\Lambda^{(ks_k)^*}$. Without any restriction, one can simple take:

$$\Lambda^{(ks_k)^*} = m_{(ks_k)^*} - \bar{z}_m^{(ks_k)^*}$$

STRANGE will explore the consequences of such a relaxation for the decision-maker be integrating the notion of "efficiency projection", developed by Winkels [12,13]. The following one parametric LP problem P_{m+1} is considered

$$P_{m+1}: \begin{cases} \min M.\ \delta - \sum_{k=1}^{K+1} \epsilon_k & \\ \sum_{s_k=1}^{S_k} p_{s_k}^{(k)} (c^{ks_k} . Z - M^{ks_k}) \pi^{ks_k} \leq \delta - \epsilon_k & k=1,\dots,K+1 \\ c^{(ks_k)*} . Z = \bar{z}^{(ks_k)*} + \lambda \Lambda^{(ks_k)*} & 0 \leq \lambda \leq 1 \\ Z \in \bar{D}_{m+1}(Z) & (10) \\ \epsilon_k \geq 0 & k=1,\dots,K+1 \end{cases}$$

where $\bar{D}_{m+1}$ is defined by:

. $\bar{D}_2(Z) = D_1(Z)$ for initialization

. $\bar{D}_{m+2}(Z) = \bar{D}_{m+1}(Z) \cap \{Z | z^{(ks_k)*}(Z) \leq z^{(ks_k)*}(\tilde{Z}_{m+1})\}$ (11)

for $m \geq 1$, after compromise $\tilde{Z}_{m+1}$ has been determined.

Using a dual simplex technique, we determine the sequence of bounds λ_i $(i = 1,\dots,\nu)$, with $0 \leq \lambda_1 \leq \leq \dots \leq \lambda_\nu = 1$, corresponding to the stable intervals for the optimal bases of the problem.

Let us call $Z_{m+1}(\lambda)$, the optimal solution of P_{m+1}. For each criterion $(1t_1)$, we determine the function $z^{1t_1}(Z_{m+1}(\lambda))$, which is a piecewise linear function, as shown in the Fig. 3.

So the decision-maker is provided, with a complete graphical analysis of the relaxation of criterion $(ks_k)*$.

Remark:

(i) It might be more comvenient for the decision-maker to examine the variation of all the criteria (lt_l), w.r.t. parameter λ. In this case, it is better to consider the relative values:

$$\frac{z^{lt}(Z_{m+1}(\lambda)) - M^{lt}}{m^{lt} - M^{lt}}$$

which are all included in the interval $[0,1]$.

(ii) Such graphics can also be easily determined for the mean values $\bar{z}^k(Z_{m+1}(\lambda))$ and for the confidence level $1 - \alpha_m^k(\lambda)$.

After analysis of these graphics, the decision-maker must then indicate an acceptable level $\tilde{\lambda}$ of relaxation, so that the new compromise $\tilde{Z}_{m+1}$ is obtained

$$\tilde{Z}_{m+1} = Z_{m+1}(\tilde{\lambda})$$

In one sense, $\tilde{\lambda}$ corresponds to the maximum level of relaxation accepted with respect to the value

$z_m^{(ks_k)*}$. In the possible next interactive phases, this level is at least imposed for the new compromises, so that improvements are still possible.

At each new interactive phase[+], the decision-maker must choose a different criterion $(ks_k)^*$ than in the preceding phases, the procedure thus comes to an end, after a maximum

of $\sum_{s_K=1}^{K+1} s_k$ interactive phases.

Remark:

In some applications, it can be convenient for the decision-maker to impose relaxation with regards to the mean value of a particular objective k^*; in that case, relation (10) is replaced by:

$$\bar{z}^{k^*}(Z) = \bar{z}^{k^*} + \lambda\, \Lambda^{k^*}$$

where Λ^{k^*} is an upper limit of the increase of $\bar{z}^{k^*}$.

This is the case whenever correlations between scenarios are such that relaxing the individual criterion ks'_k makes all other criteria ks'_k move at the same time. For instance, in economic objectives, unit costs pertaining to different scenarios have a limited range of variation, and the corresponding cost functions are not completely independent.

+When a criterion $(ks_k)^*$ is chosen to be relaxed, its weight $\pi^{(ks_k)^*}$ is set to zero for the following steps of the study.

<u>Example</u>:

<u>First interactive phase</u>:

. The three pieces of information resulting from the first compromise are presented to the D.M. as follows:

ks_k	$\bar{z}^{ks_k}$	$[M^{ks_k}, m^{ks_k}]$
11	10.49	[1,25]
12	14.46	[3.625,27.5]
13	18.43	[5.25,30]
21	4.82	[1,9.25]
22	6.80	[3.0625,10.0625]
31	2.12	[1, 4]

k	$\bar{z}_1^k$	$1 - \alpha_1^k$
1	14.46	0.75
2	6.31	0.25

. The D.M. indicates:

$$(ks_k)^* = (1,1) \qquad \Lambda^{(ks_k)^*} = 14.51 = m_{(ks_k)^*} - \bar{z}^{(ks_k)^*}$$

. The parametric LP problem P_2 (equation (10)) is solved, and a graphic representation of the relative variations of all the criteria is given in Fig. 4; the absolute values are given in the following table:

	z^{11}	z^{12}	z^{13}	z^{21}	z^{22}	z^{31}
$\lambda_0 = 0.000$	10.491	14.460	18.428	4.821	6.805	2.112
$\lambda_1 = 0.107$	12.040	16.026	20.013	4.354	6.347	1.982
$\lambda_2 = 0.168$	12.931	16.325	19.718	4.588	6.285	1.300
$\lambda_3 = 0.249$	14.107	16.718	19.329	4.897	6.203	1.000
$\lambda_4 = 0.463$	17.203	17.754	18.305	5.711	5.986	1.000
$\lambda_5 = 1.000$	25.000	27.500	30.000	1.813	3.063	1.000

. The D.M. chooses the level of relaxation λ_1 corresponding to the following set of data.

ks_k	$\bar{z}_2^{ks_k}$
11	12.04
12	16.03
13	20.01
21	4.35
22	6.35
31	1.98

k	$\bar{z}_2^k$	$1 - \alpha_2^k$
1	16.03	0.75
2	5.84	0.25

The corresponding non-vanishing variables are (see Fig. 2):

$x_1 = 4.82$ $\quad x_2 = 3.15$ $\quad v_2 = 0.73$

$w^3 = 2.27$ $\quad w^4 = 5.27$

<u>Second interactive phase</u>:

. From the previous step, the D.M. now indicates the next candidate for relaxation:

$$(ks_k)^* = (2,1)$$

. The parametric LP problem P_3 is solved as shown in Fig. 5, where a relative scale for all the criteria has been chosen; the absolute values are given in the following table:

	z^{11}	z^{12}	z^{13}	z^{21}	z^{22}	z^{31}
$\lambda'_0 = 0.000$	12.040	16.026	20.013	4.354	6.347	1.982
$\lambda'_1 = 0.164$	10.263	13.905	17.547	5.159	6.980	1.816
$\lambda'_2 = 0.209$	10.833	14.016	17.199	5.375	6.967	1.300
$\lambda'_3 = 0.286$	11.828	14.211	16.594	5.752	6.943	1.000
$\lambda'_4 = 0.302$	12.039	14.252	16.465	5.832	6.938	1.000
$\lambda'_5 = 0.616$	12.039	12.409	12.779	7.368	7.553	1.000
$\lambda'_6 = 0.962$	3.000	4.500	6.000	9.063	9.813	1.000
$\lambda'_7 = 1.000$	2.000	3.625	5.250	9.250	10.063	1.075

. D.M. choses the level of relaxation λ'_1 corresponding to the following data:

ks_k	$\bar{z}_3^{ks_k}$
11	10.26
12	13.90
13	17.55
21	5.16
22	6.98
31	1.82

k	$\bar{z}_3^k$	$1 - \alpha_3^k$
1	13.90	0.75
2	6.53	0.25

The corresponding non-vanishing variables are (see Fig. 2):

$$x_1 = 4.07 \qquad x_2 = 3.22$$
$$v^2 = 1.28$$
$$w^3 = 1.72 \qquad w^4 = 4.72$$

A third relaxation phase is carried out on the third objective. As can be seen in Fig. 6, this relaxation does not bring any significant improvement of the other criteria. The DM decides to keep the solution obtained during the second relaxation step.

CONCLUSIONS

The MOSLP approach used in the strategy code STRANGE has been described in detail and illustrated with a simple numerical example. As a decision aid, it has already been used in real world problems [6,7,11] and might therefore claim to be useful and simple to understand from the point of view of a D.M.

Its basic approach is STEM, although it largely differs from it mainly in the following respects:

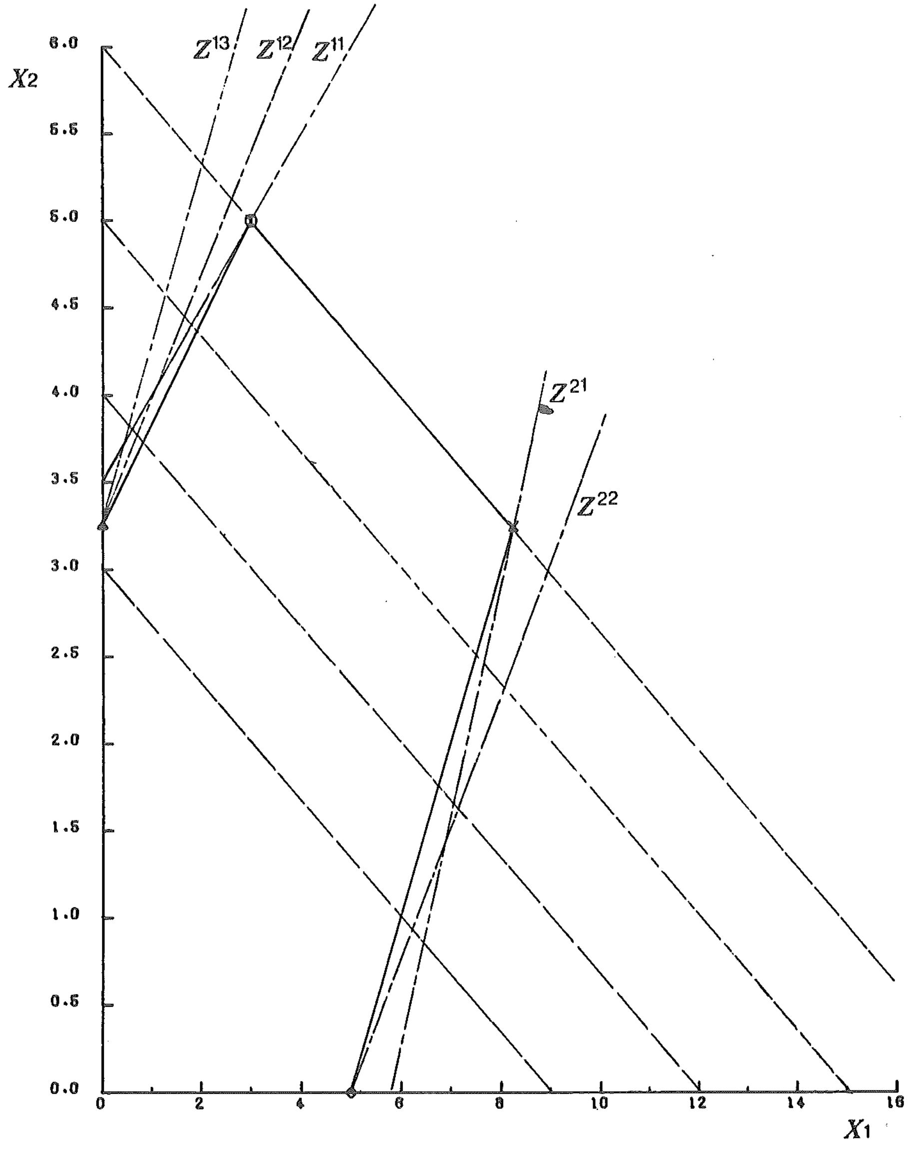
- it provides efficient solutions;

- rather than giving isolated results difficult to interpret, it provides the D.M. with families of efficient solutions and visualized functional dependence between criteria. This leaves more space for successive guesses and trials from the D.M.;

- it includes stochastic aspects.

Regarding the last point, the definition of scenarios in the resolution algorithm seems to be the easiest and most understandable way of describing future uncertainties. Other approaches have their merits, but are nevertheless not very easy to use in large scale problems such as those encountered in energy planning.

Moreover, it looks as if the STRANGE approach also has potential for mixed variable problems on which our current effort is centred.



OBJECTIVES - INDIVIDUAL OPTIMISATIONS

FIG. 1

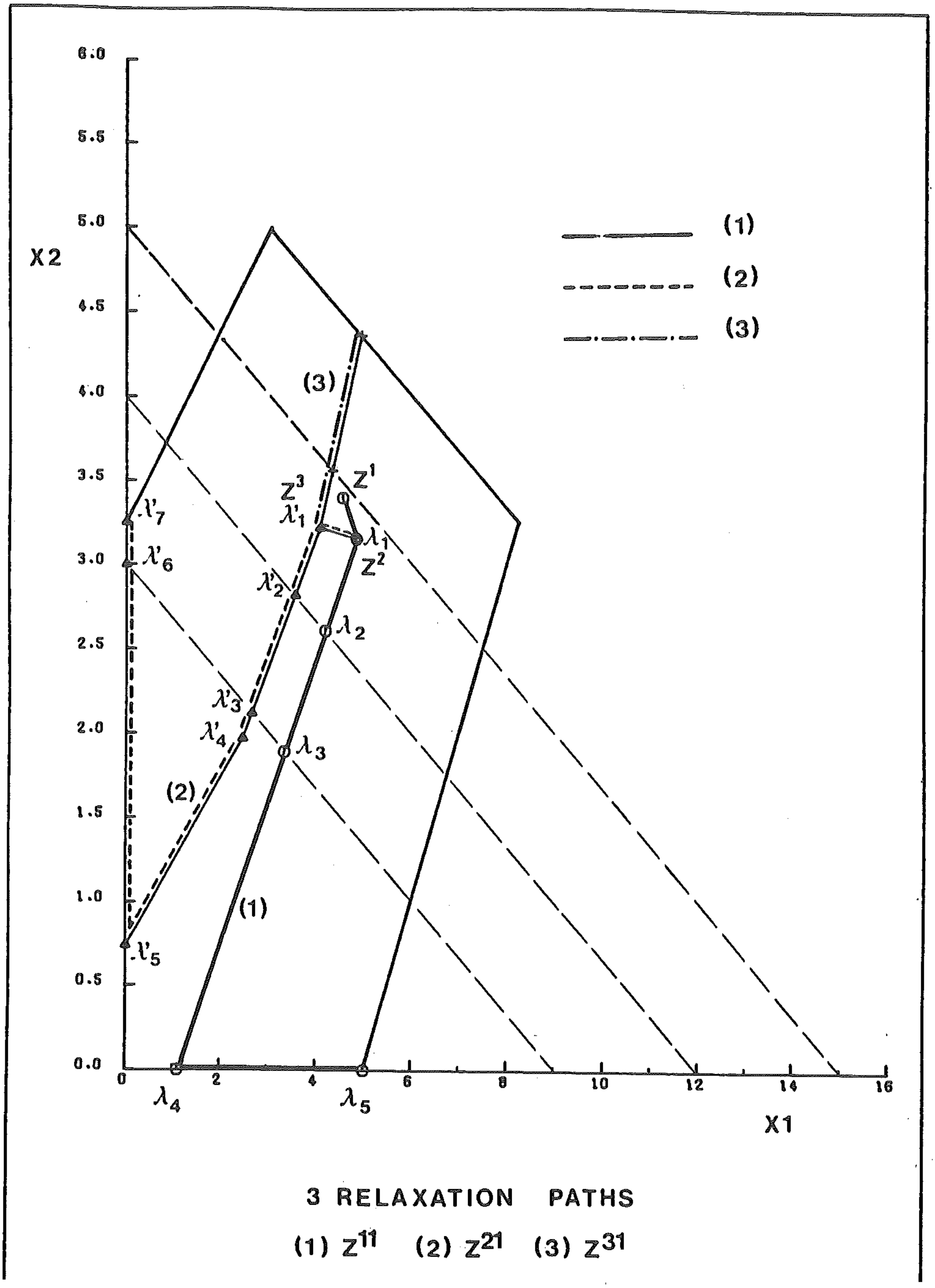
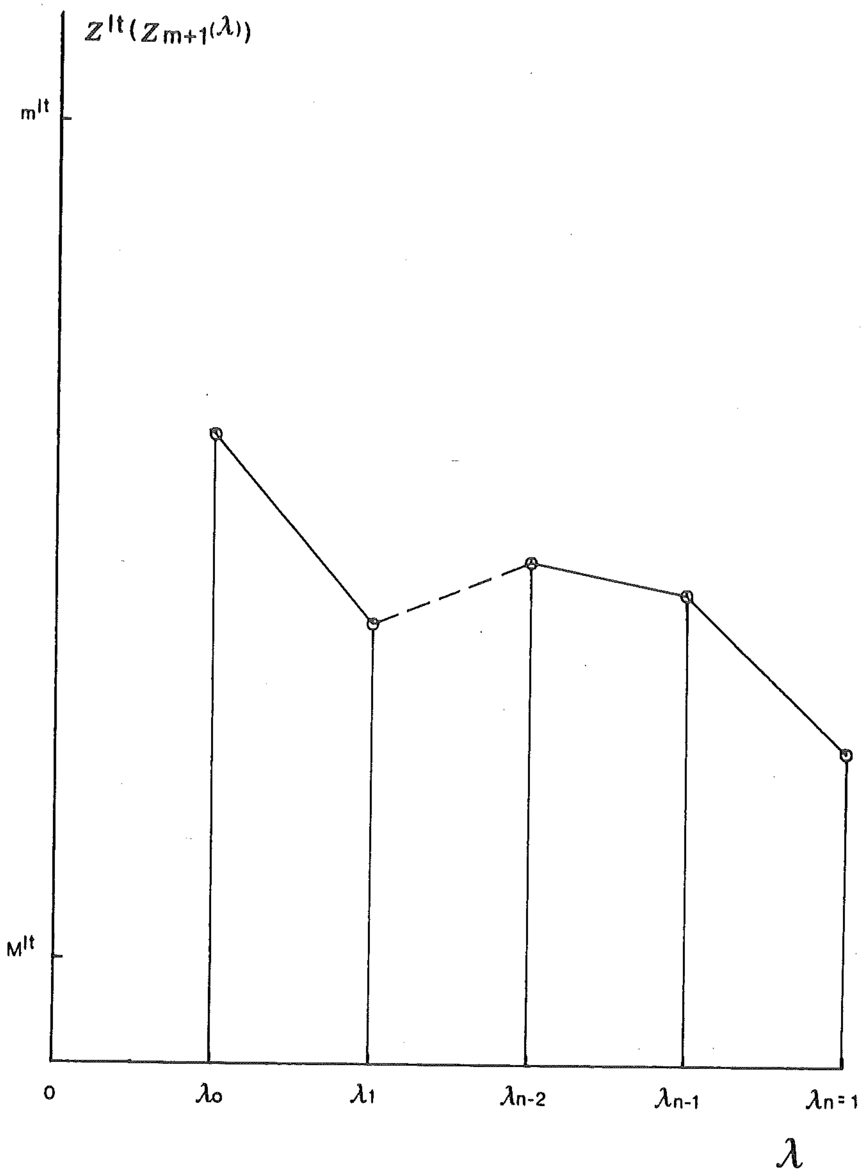
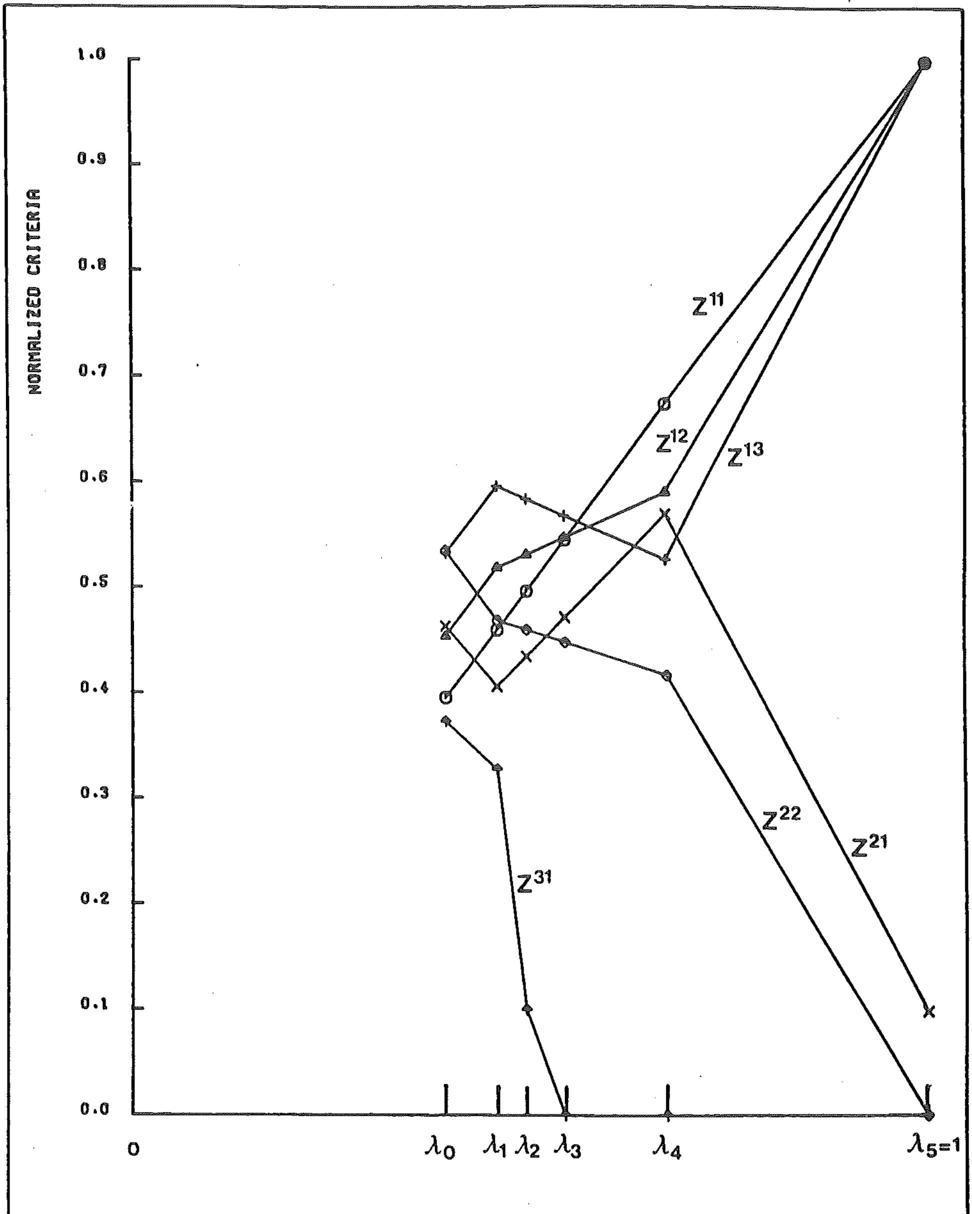


FIG. 2



OPTIMAL CHANGES OF BASE

FIG. 3



FIRST INTERACTIVE STEP

FIG.4

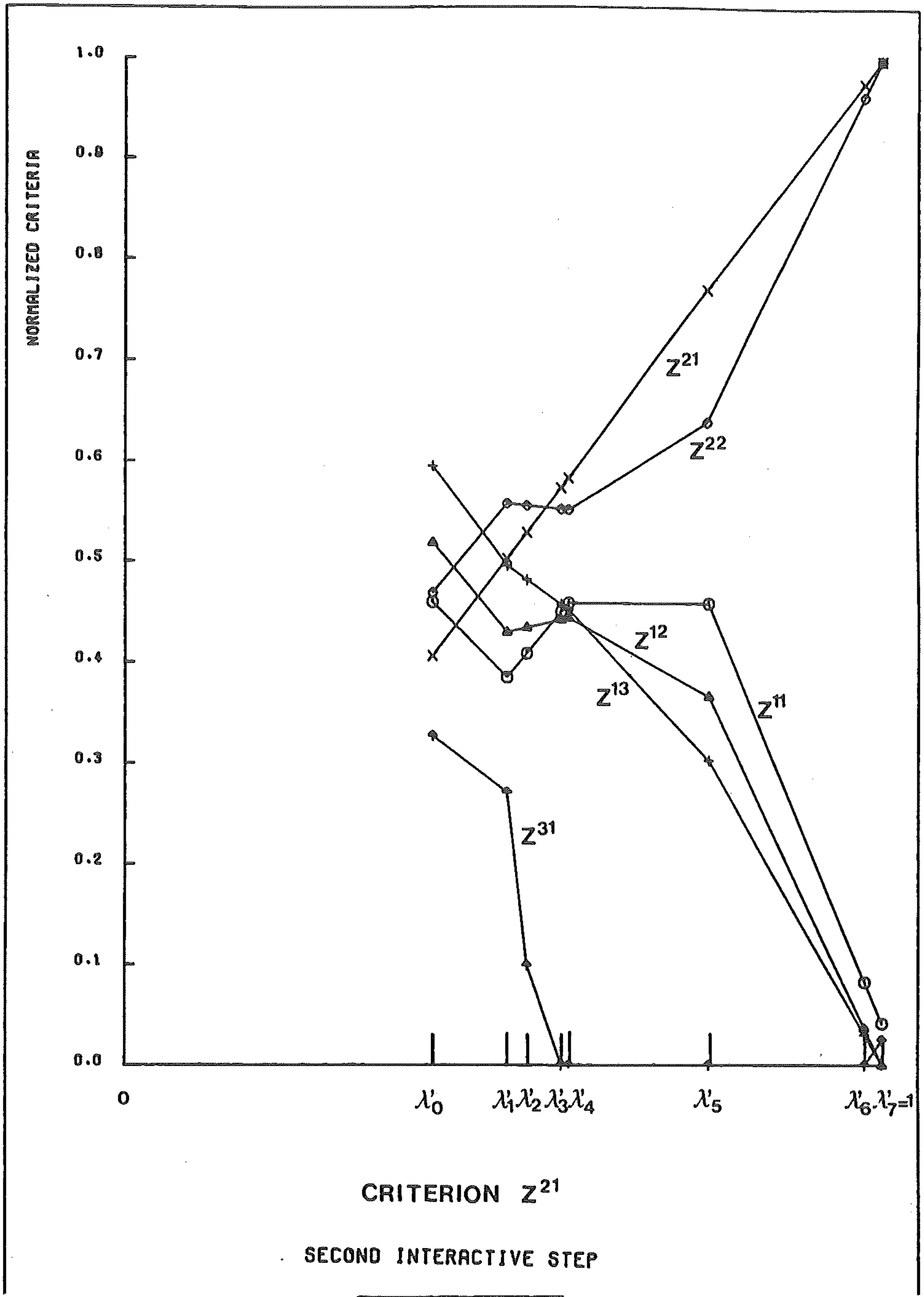

NORMALIZED CRITERIA
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Z21
Z22
Z12
Z13
Z11
Z31
0
λ'0
λ'1 λ'2 λ'3 λ'4
λ'5
λ'6 λ'7=1
CRITERION Z21
SECOND INTERACTIVE STEP

FIG. 5

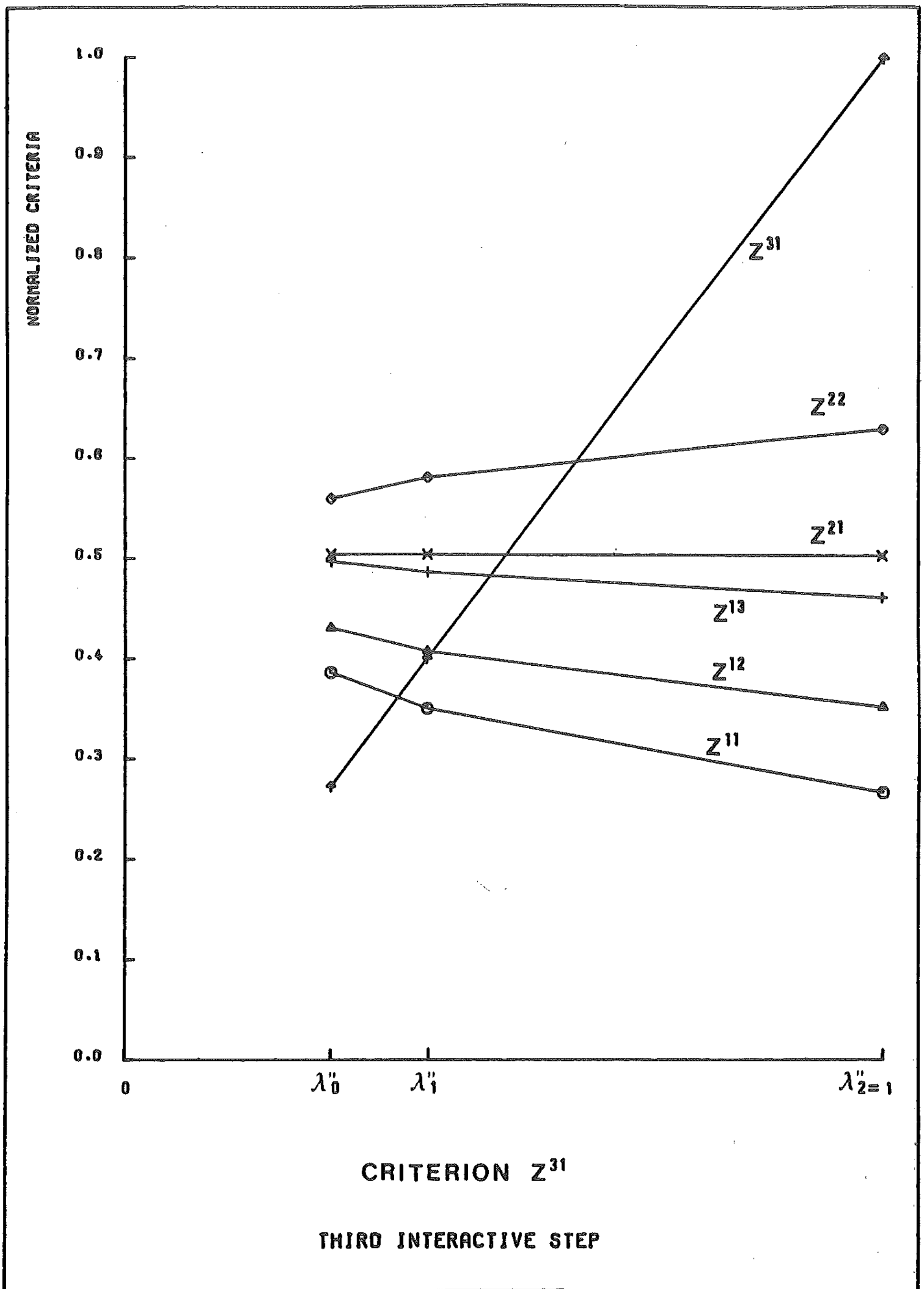

NORMALIZED CRITERIA
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
λ''_0
λ''_1
$\lambda''_{2=1}$
Z^{31}
Z^{22}
Z^{21}
Z^{13}
Z^{12}
Z^{11}
CRITERION Z^{31}
THIRD INTERACTIVE STEP

FIG. 6

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A NOTE ON THE PAY-OFF MATRIX IN MULTIPLE OBJECTIVE PROGRAMMING

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ABSTRACT

The pay-off matrix is a well-known device in multiple objective programming. It helps to understand the conflicts between different goal variables. Furthermore, the pay-off matrix constitutes the basis for the calculation of frequently used references vectors such as the ideal (utopia) vector and the nadir vector. Unfortunately, when the separate optimization of the individual goal variables does not result in unique optimal solutions, the corresponding nadir vector is not uniquely defined. Ignoring this phenomenon may have a serious effect on several multiple objective programming methods. In this paper we re(de)fine the notion of nadir vector in order to take account of the possibility of alternative optimal solutions. We also present a procedure which generates this uniquely defined nadir vector.

UTASTAR : AN ORDINAL REGRESSION METHOD FOR BUILDING ADDITIVE VALUE FUNCTIONS

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ABSTRACT

This paper presents an improved version of the UTA method performing an ordinal regression analysis by means of more powerful linear programming formulations. The ordinal variable to be analysed is a weak-order relation whereas the independent variables are criteria, i.e. quantitative and/or qualitative monotone variables. The method is illustrated by a simple numerical example. Finally, experimental results are given, demonstrating by means of three distinct indicators, the superiority of the adjustements obtained with the new method.

THE DANTZIG WOLFE DECOMPOSITION EXTENDED TO MULTI-CRITERIA LINEAR PROGRAMMING - THEOREMS, PROOFS AND ALGORITHM

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ABSTRACT

The use of a simplex method (either with Zeleny or Philips technique) to find the efficient solutions of a linear multi-objective problem rises increasing time and memory storage difficulties as problems become off large dimension.

In fact, for practical purposes, most authors share the opinion that the process should be controled with the help of other criteria, namely the decision maker´s preferences, either in a direct or indirect way.

In any case, every contribution to increase speed and efficiency of the multi-objective simplex is welcome. In this paper, the extension of the Dantzig-Wolfe decomposition to a linear multi-objective problem is presented, along with the theorems and proofs required by an efficient solution search algorithm.

LABOUR STABILITY Vs BUSINESS PROFITABILITY WITHIN AN AGRARIAN REFORM PROGRAMME IN ANDALUSIA (SPAIN): A COMPROMISE PROGRAMMING APPLICATION

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ABSTRACT

In this paper a real problem related to the implementation of the 1984 Agrarian Reform for Andalusia (Spain) is analysed. The problem lies in the degree of conflict between one of the main objectives of the agrarian reform programme: to provide stable employment, and perhaps the main objective of the labourers associated into cooperatives established by the Agrarian Reform: maximisation of business profitability. A compromise between both conflicting objectives in this real case is established by resorting to multiobjective and compromise programming techniques.

Key words: agrarian reform, compromise programming, multiobjective programming, rural employment.

STRANGE: AN INTERACTIVE METHOD FOR MULTI-OBJECTIVE LINEAR PROGRAMMING UNDER UNCERTAINTY

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ABSTRACT

In the field of investment planning within a time horizon, problems typically involve multiple decision objectives, and basic data are uncertain. In a large number of cases, these decision problems can be written as linear programming systems in which time dependent uncertainties affect the coefficients of objectives and the RHS of the constraints. Given the possibility of defining plausible scenarios on basic data, discrete sets of such coefficients are given, each with its subjective probability of occurrence. The corresponding structure is then characteristic for Multi-Objective Stochastic Linear programming (MOSLP).

In the paper, an interactive procedure is described to obtain a best compromise for such a MOSLP problem. This algorithm, called STRANGE, extends the STEM method to take the random aspects into account. It involves in particular, the concepts of stochastic programming with recourse. In its interactive steps, the efficiency projection techniques are used to provide the decision-maker with detailed graphical information on efficient solution families.

As an illustration of the successive steps, a didatic example is solved in some detail.

INDICE

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