Models for Optimal Survivable Routing with a Minimum Number of Hops: Comparing Disaggregated with Aggregated Models

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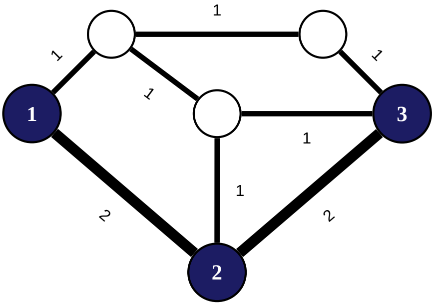
Workshop Investigação Operacional nas Telecomunicações
APDIO
Coimbra, Novembro 2011

Summary

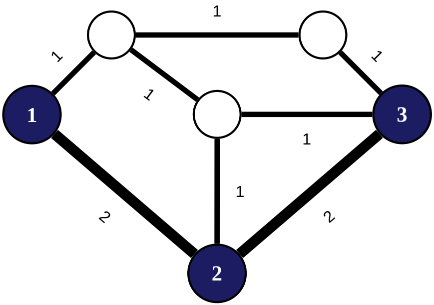
Traffic Engineering Problem



Given an undirected graph $N = (X, U)$, with:

- **capacities $b_e, e \in U$;**
- **a set of commodities with origin/destination nodes $p, q \in S \subset X$ ($S = \{1, 2, 3\}$);**
- **a traffic demand matrix $R = [r_{pq}], p, q \in S$;**

Traffic Engineering Problem



How to route the traffic demand matrix $R = [r_{pq}]$ over N?

Complying with ($\forall p, q \in S$):

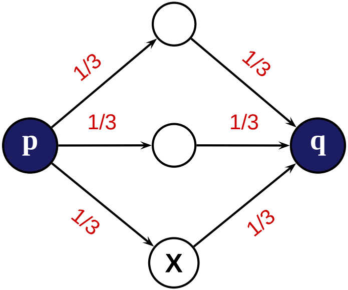
- **Node-disjointness (D)**;
- **Hop-constraints (H)**;

- **Installed bandwidth ($b_e, e \in E$)**;
- **Survivability mechanism:** Path Diversity or Path Protection.

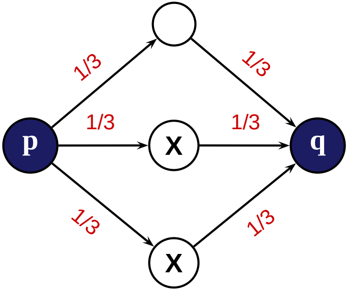
Traffic Engineering Problem

Path Diversity

for all $p, q \in S$, each demand is equally split by the D node-disjoint paths, i.e., $\Delta = D$



$D = 3, t_{pq} = 1, n = 1$
protected demand: 66%

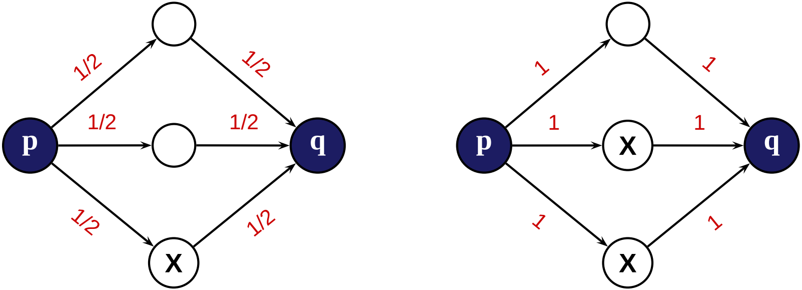


$D = 3, t_{pq} = 1, n = 2$
protected demand: 33%

Traffic Engineering Problem

Path Protection

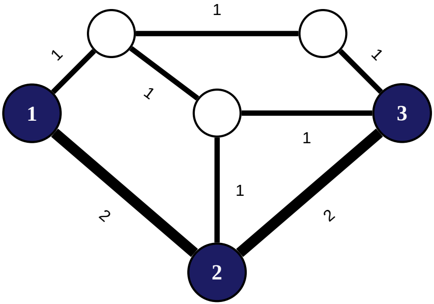
for all $p, q \in S$, if n paths fail, the $D - n$ remaining ones must accommodate total demand, i.e., $\Delta = D - n$



$D = 3, t_{pq} = 1, n = 1$
protected demand: 100%

$D = 3, t_{pq} = 1, n = 2$
protected demand: 100%

Traffic Engineering Problem



How to route the traffic demand matrix $R = [r_{pq}]$ over N?

While **minimizing**:

- **the average number of hops**

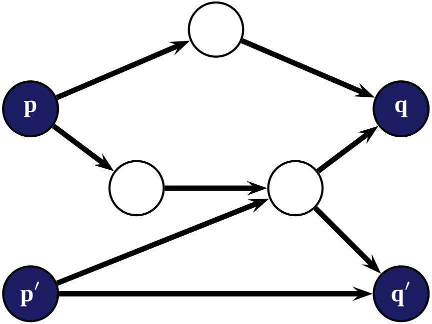
  or

- **the maximum number of hops**

of the Δ minimum hop routing paths between any pair $p, q \in S$, with $1 \leq \Delta \leq D$.

Traffic Engineering Problem

$D = 2, H = 3$



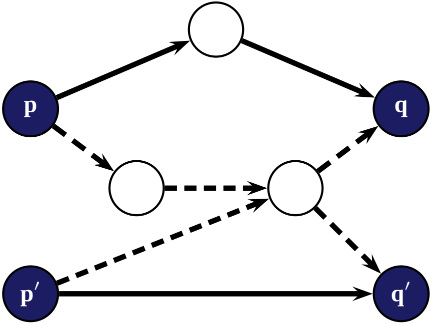
Path Diversity

$\Delta = D = 2$

Min-Ave	Min-Max
$((2+3)+(2+1))/4$	3

Traffic Engineering Problem

$D = 2, H = 3$



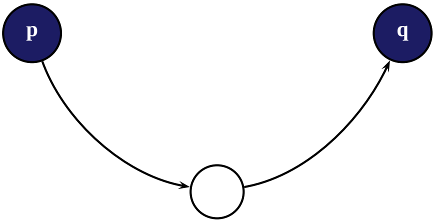
Path Protection ($n = 1$)

$\Delta = D - 1 = 1$

Min-Ave	Min-Max
$(2+1)/2$	2

(p,q)-*H*-*path*

$N = (X, U)$



(p,q)-2-*path*

$\forall p, q \in S$

a **(p,q)-*H*-*path*** is a sequence of arcs from p to q with at most H hops

Variables Definition (Disaggregated)

$$z_{ij}^{h,pq,d} = \begin{cases} 1, & \text{if the } d^{th}\ (p,q)\text{-}H\text{-}path \\ & \text{traverses edge } \{i,j\} \text{ from } i \text{ to } j \\ & \text{in the } h^{th} \text{ position} \\ 0, & \text{otherwise} \end{cases},$$

$$(i,j) \in A \cup (q,q); h = 1, ..., H; d = 1, ..., D; p, q \in S.$$

Modeling the Constraints

Modeling D (p,q)-H-$paths$, for all $p, q \in S$ **(2.1)**

$$\sum_{j:(p,j)\in A} z_{pj}^{1,pq,d} = 1, d = 1, \ldots, D; p, q \in S$$

$$\sum_{j:(i,j)\in A} z_{ij}^{h+1,pq,d} - \sum_{j:(j,i)\in A} z_{ji}^{h,pq,d} = 0,\ i \neq p; h = 1, \ldots, H-1; d = 1, \ldots, D; p, q \in S$$

$$\sum_{j:(j,q)\in A} z_{jq}^{H,pq,d} = 1,\ d = 1, \ldots, D; p, q \in S$$

Modeling node disjointness, for all $p, q \in S$ **(2.2)**

$$\sum_{i:(i,j)\in A} \sum_{h=1,..,H} \sum_{d=1,\ldots,D} z_{ij}^{h,pq,d} \leq 1,\ j \in X \setminus \{p, q\}; p, q \in S$$

Complying with the installed bandwidth, for all $e \in U$ **(2.3)**

$$\sum_{p,q\in S} \beta r_{pq} \left(\sum_{h=1,..,H} \sum_{d=1,\ldots,D} \left(z_{ij}^{h,pq,d} + z_{ji}^{h,pq,d} \right) \right) \leq b_e, e = \{i, j\} \in U$$

Minimizing the Average and Minimizing the Maximum Number of Hops

D-Ave-Arc

$$Min \sum_{d=1,\ldots,\Delta} \sum_{p,q\in S} \sum_{(i,j)\in A} \sum_{h=1,\ldots,H} z_{ij}^{h,pq,d}$$

subject to **(2.1)**, **(2.2)**, **(2.3)**, **(2.4)**.

Minimizing the Average and Minimizing the Maximum Number of Hops

$V =$ number of hops of the worst path among the best Δ paths for all commodities

D-Ave-Arc

$$Min \sum_{d=1,\dots,\Delta} \sum_{p,q \in S} \sum_{(i,j)\in A} \sum_{h=1,\dots,H} z_{ij}^{h,pq,d}$$

subject to **(2.1)**, **(2.2)**, **(2.3)**, **(2.4)**.

D-Max-Arc

$$Min\ V$$

subject to **(2.1)**, **(2.2)**, **(2.3)**, **(2.4)**

$$\sum_{(i,j)\in A} \sum_{h=1,\dots,H} z_{ij}^{h,pq,d} \le V,\ p,q \in S; d = 1, \dots, \Delta$$

$V \ge 0$ and integer.

Variables Definition (Aggregated)

$w_{ij}^{h,pq} =$ number of (p,q)-H-$paths$ traversing edge $\{i,j\}$ from i to j in the h^{th} position,

$(i,j) \in A \cup (q,q); h = 1, ..., H; p, q \in S.$

Modeling the Constraints

Modeling D (p,q)-H-$paths$, for all $p,q \in S$ **(3.1)**

$$\sum_{j:(p,j)\in A} w_{pj}^{1,pq} = D,\; p,q \in S$$

$$\sum_{j:(i,j)\in A} w_{ij}^{h+1,pq} - \sum_{j:(j,i)\in A} w_{ji}^{h,pq} = 0,\; i \neq p; h = 1, \ldots, H-1; p,q \in S$$

$$\sum_{j:(j,q)\in A} w_{jq}^{H,pq} = D,\; p,q \in S$$

Modeling node disjointness, for all $p,q \in S$ **(3.2)**

$$\sum_{i:\{i,j\}\in U} \sum_{h=1,\ldots,H} w_{ij}^{h,pq} \leq 1, j \in X \setminus \{p,q\}; p,q \in S$$

Complying with the installed bandwidth, for all $e \in U$ **(3.3)**

$$\sum_{p,q\in S} \beta r_{pq} \left(\sum_{h=1,\ldots,H} \left(w_{ij}^{h,pq} + w_{ji}^{h,pq} \right) \right) \leq b_e, e = \{i,j\} \in U$$

Minimizing the Average Number of Hops

For a given h and $p, q \in S$:

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} = \text{number of arcs in the } h^{th} \text{ position of the } D \text{ } (p,q)\text{-}H\text{-}paths$$

$U^{h,pq} =$ number of “interesting” arcs (arcs included in the best Δ paths) in the h^{th} position

Minimizing the Average Number of Hops

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$U^{h,pq}$ = number of "interesting" arcs (arcs included in the best Δ paths) in the h^{th} position

Minimizing the average number of hops of all the Δ best paths of every commodity:

$$Min \sum_{p,q\in S} \sum_{h=1,\ldots,H} U^{h,pq}$$

Minimizing the Average Number of Hops

For a given h and $p, q \in S$:

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} = \text{number of arcs in the } h^{th} \text{ position of the } D \text{ } (p,q)\text{-}H\text{-}paths$$

$$U^{h,pq} = \text{number of “interesting” arcs (arcs included in the best } \Delta \text{ paths) in the } h^{th} \text{ position}$$

Minimizing the average number of hops of all the Δ best paths of every commodity:

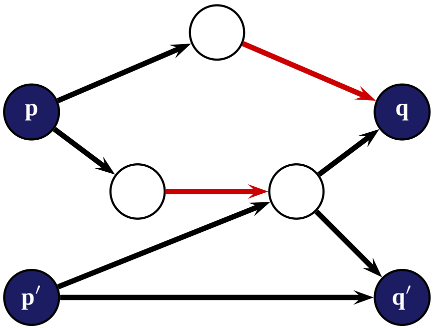
$$Min \sum_{p,q\in S} \sum_{h=1,\dots,H} U^{h,pq}$$

For a given h and $p, q \in S$, the number of interesting arcs
under Path Diversity: all ($D - \Delta = 0$)
under Path Protection: all$-n$ ($D - \Delta = n$)

$$U^{h,pq} \geq \sum_{(i,j)\in A} w_{ij}^{h,pq} - (D - \Delta)$$

Motivation

$D = 2, H = 3$



Path Diversity
$\Delta = D = 2$

For $h = 2$,

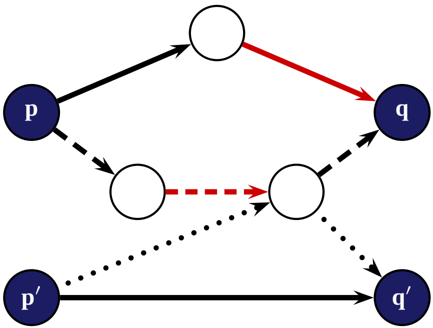
$$\sum_{(i,j)\in A} w_{ij}^{2,pq} = 2$$

$$\Downarrow \qquad U^{h,pq} \geq \sum_{(i,j)\in A} w_{ij}^{h,pq} - (D - \Delta)$$

$\exists$ two relevant paths with at least $h = 2$ arcs

Motivation

$D = 2, H = 3$



Path Protection ($n = 1$)
$\Delta = D - 1 = 1$

For $h = 2$,

$$\sum_{(i,j) \in A} w_{ij}^{2,pq} = 2$$

$$\Downarrow \qquad U^{h,pq} \geq \sum_{(i,j) \in A} w_{ij}^{h,pq} - (D - \Delta)$$

$\exists$ one relevant path
with at least $h = 2$ arcs

Minimizing the Average Number of Hops

A-Ave-Arc

$$Min \sum_{p,q \in S} \sum_{h=1,\ldots,H} U^{h,pq}$$

subject to **(3.1)**, **(3.2)**, **(3.3)**, **(3.4)**

$$U^{h,pq} \geq \sum_{(i,j) \in A} w_{ij}^{h,pq} - (D - \Delta), \; p, q \in S; h = 1, \ldots, H$$

$$U^{h,pq} \in \{0, 1, \ldots, \Delta\}, \; p, q \in S; h = 1, \ldots, H.$$

Min-Average Case

Result 1

i) When $D = \Delta$, $v(D\text{-}Ave\text{-}Arc_L) = v(A\text{-}Ave\text{-}Arc_L)$

ii) When $D > \Delta$, it is still open whether

a) $v(D\text{-}Ave\text{-}Arc_L) = v(A\text{-}Ave\text{-}Arc_L)$ or

b) $v(D\text{-}Ave\text{-}Arc_L) \geq v(A\text{-}Ave\text{-}Arc_L)$ and there are instances for which the inequality is strict.

Minimizing the Maximum Number of Hops

For a given h:

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} = \text{number of arcs in the } h^{th} \text{ position of the } D\ (p,q)\text{-}H\text{-}paths, p.q \in S$$

$$V^h = \begin{cases} 1, & \text{if the number of arcs in position } h, \\ & \text{for at least one of the paths of all commodities,} \\ & \text{is} \geq (D - \Delta) + 1 \\ \\ 0, & \text{otherwise} \end{cases}$$

Minimizing the Maximum Number of Hops

For a given h:

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} = \text{number of arcs in the } h^{th} \text{ position of the } D\ (p,q)\text{-}H\text{-}paths,\ p.q \in S$$

$$V^h = \begin{cases} 1, & \text{if the number of arcs in position } h, \\ & \text{for at least one of the paths of all commodities,} \\ & \text{is } \geq (D - \Delta) + 1 \\ & \\ 0, & \text{otherwise} \end{cases}$$

Minimizing the maximum number of hops of the Δ best paths of every commodity:

$$Min \sum_{h=1,\ldots,H} V^h$$

Minimizing the Maximum Number of Hops

For a given h:

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} = \text{number of arcs in the } h^{th} \text{ position of the } D \ (p,q)\text{-}H\text{-}paths, p.q \in S$$

$$V^h = \begin{cases} 1, & \text{if the number of arcs in position } h, \\ & \text{for at least one of the paths of all commodities,} \\ & \text{is } \geq (D-\Delta)+1 \\ 0, & \text{otherwise} \end{cases}$$

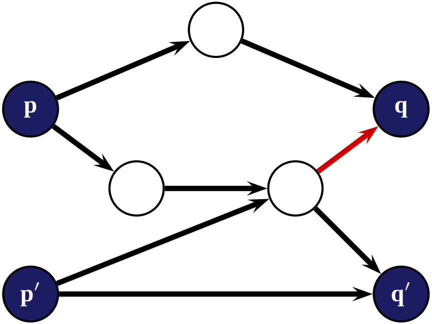
Minimizing the maximum number of hops of all the Δ best paths of every commodity:

$$Min \sum_{h=1,\dots,H} V^h$$

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} - (D-\Delta) \leq V^h \times \Delta, \; p,q \in S; h = 1,\dots,H$$

Motivation

$D = 2, H = 3$



Path Diversity
$\Delta = D = 2$

For $h = 3$,

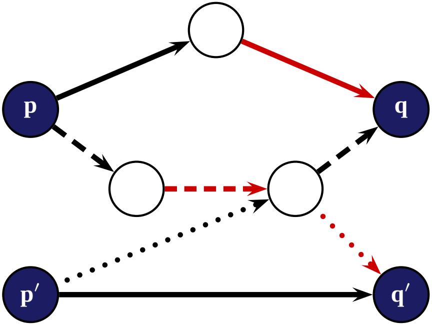
$$\sum_{(i,j)\in A} w_{ij}^{3,pq} = 1$$

$$\Downarrow \qquad \sum_{(i,j)\in A} w_{ij}^{h,pq} - (D - \Delta) \leq V^h \times \Delta$$

$\exists$ one relevant path with at least $h = 3$ arcs

Motivation

$D = 2, H = 3$



Path Protection ($n = 1$)

$\Delta = D - 1 = 1$

For $h = 2$,

$$\sum_{(i,j)\in A} w_{ij}^{2,pq} = 2$$

$$\Downarrow \qquad \sum_{(i,j)\in A} w_{ij}^{h,pq} - (D - \Delta) \leq V^h \times \Delta$$

$\exists$ one relevant path with at least $h = 2$ arcs

Minimizing the Maximum Number of Hops

A-Max-Arc

$$Min \sum_{h=1,\dots,H} V^h$$

subject to **(3.1)**, **(3.2)**, **(3.3)**, **(3.4)**

$$\sum_{(i,j)\in A} w_{ij}^{h,pq} - (D - \Delta) \leq V^h \times \Delta,\ p, q \in S; h = 1, \dots, H$$

$$V^h \in \{0, 1\},\ h = 1, \dots, H.$$

Min-Max Case

The D-Max-Arc and A-Max-Arc models are not comparable

D-Max-Arc

$Min\ V$

subject to **(2.1)**, **(2.2)**, **(2.3)**, **(2.4)**

$$\sum_{(i,j)\in A}\sum_{h=1,\dots,H} z_{ij}^{h,pq,d} \leq V, \quad p, q \in S; d = 1, \dots, \Delta$$

$V \geq 0$ and integer

Min-Max Case

The D-Max-Arc and A-Max-Arc models are not comparable

D-Max-Arc

$Min\ V$

subject to **(2.1)**, **(2.2)**, **(2.3)**, **(2.4)**

$$\sum_{(i,j)\in A} \sum_{h=1,\ldots,H} z_{ij}^{h,pq,d} \leq V,$$
$$p, q \in S; d = 1, \ldots, \Delta$$

$V \geq 0$ and integer

$$\sum_{(i,j)\in A} z_{ij}^{h,pq,d} \leq V^h,$$
$$h = 1, \ldots H; p, q \in S; d = 1, \ldots, \Delta$$

$V^h \in \{0, 1\}, h = 1, \ldots, H.$ ← from A-Max-Arc

Min-Max Case

The D-Max-Arc and A-Max-Arc models are not comparable

~~D-Max-Arc~~ SD-Max-Arc

~~$Min\ V$~~ $Min \sum_{h=1,\ldots,H} V^h$

subject to **(2.1)**, **(2.2)**, **(2.3)**, **(2.4)**

~~$\sum_{(i,j)\in A} \sum_{h=1,\ldots,H} z_{ij}^{h,pq,d} \leq V,$~~
~~$p, q \in S; d = 1, \ldots, \Delta$~~

~~$V \geq 0$ and integer~~

$$\sum_{(i,j)\in A} z_{ij}^{h,pq,d} \leq V^h,$$
$$h = 1, \ldots H; p, q \in S; d = 1, \ldots, \Delta$$

$$V^h \in \{0, 1\}, h = 1, \ldots, H.$$ ← from A-Max-Arc

$$v(SD\text{-}Max\text{-}Arc_L)$$

$$\geq$$

$$v(D\text{-}Max\text{-}Arc_L)$$

and there are instances for which the inequality is strict

Min-Max Case

Result 2

i) When $D = \Delta$, $v(SD\text{-}Max\text{-}Arc_L) = v(A\text{-}Max\text{-}Arc_L)$

ii) When $D > \Delta$, it is still open whether

a) $v(SD\text{-}Max\text{-}Arc_L) = v(A\text{-}Max\text{-}Arc_L)$ or

b) $v(SD\text{-}Max\text{-}Arc_L) \geq v(A\text{-}Max\text{-}Arc_L)$ and there are instances for which the inequality is strict.

Computational Results - Min-Ave Case

Average LP gaps and CPU times (s)

Path Diversity ($\Delta = D$)					
$\mathbf{D = 2}$		$\mathbf{D = 3}$		$\mathbf{D = 4}$	
D-Ave-Arc	A-Ave-Arc	D-Ave-Arc	A-Ave-Arc	D-Ave-Arc	A-Ave-Arc
1.9% (0.02)	1.9% (0.01)	2.1% (0.03)	2.1% (0.01)	1.5% (0.04)	1.5% (0.01)

Path Protection ($\Delta = D - 1$)					
$\mathbf{D = 2}$		$\mathbf{D = 3}$		$\mathbf{D = 4}$	
D-Ave-Arc	A-Ave-Arc	D-Ave-Arc	A-Ave-Arc	D-Ave-Arc	A-Ave-Arc
0.2% (0.01)	0.2% (0.01)	1.4% (0.03)	1.4% (0.01)	1.2% (0.04)	1.2% (0.02)

Path Protection ($\Delta = D - 2$)			
$\mathbf{D = 3}$		$\mathbf{D = 4}$	
D-Ave-Arc	A-Ave-Arc	D-Ave-Arc	A-Ave-Arc
0% (0.03)	0% (0.02)	0% (0.04)	0% (0.01)

Computational Results - Min-Max Case

Average LP gaps and CPU times (s)

Path Diversity ($\Delta = D$)					
$\mathbf{D=2}$		$\mathbf{D=3}$		$\mathbf{D=4}$	
SD-Max-Arc	A-Max-Arc	SD-Max-Arc	A-Max-Arc	SD-Max-Arc	A-Max-Arc
16.7% (0.01)	16.7% (0.01)	21.5% (0.03)	21.5% (0.01)	25% (0.05)	25% (0.01)
16.7% (0.02)		27.7% (0.03)			

Path Protection ($\Delta = D - 1$)					
$\mathbf{D=2}$		$\mathbf{D=3}$		$\mathbf{D=4}$	
SD-Max-Arc	A-Max-Arc	SD-Max-Arc	A-Max-Arc	SD-Max-Arc	A-Max-Arc
0% (0.02)	0% (0.01)	16.7% (0.04)	16.7% (0.01)	12.5% (0.05)	12.5% (0.02)
0% (0.02)		16.7% (0.03)			

Path Protection ($\Delta = D - 2$)			
$\mathbf{D=3}$		$\mathbf{D=4}$	
SD-Max-Arc	A-Max-Arc	SD-Max-Arc	A-Max-Arc
0% (0.04)	0% (0.01)	16.7% (0.05)	16.7% (0.02)

Conclusions and Future Work

- In practice, the aggregated models seem to be much more efficient;
- Close the linear programming gaps obtained in the Min-Max case;
- Test the presented models with $H = 3, 4, 5$ on different (denser) network topologies. We hope that looking at the linear programming relaxation optimal solutions will shed some light on the still open issues of results 1 and 2;