The Load Balancing Optimization of Telecommunication Networks

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Introduction

Single path routing

- Capacitated telecommunications network

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  - single path routing

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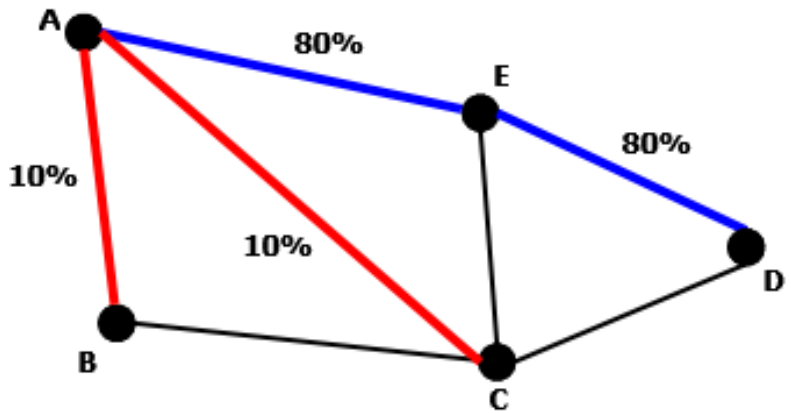
- Example:
  - Links have capacity 100 Mbps
  - Commodity A-D has demand 80 Mbps
  - Commodity B-C has demand 10 Mbps



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  - minimize the second worst link load, guaranteeing that the worst link load is optimized
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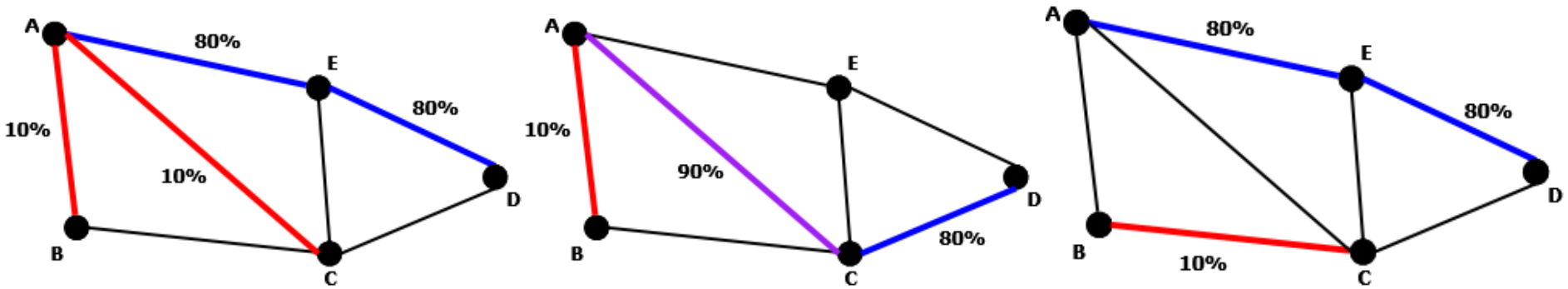

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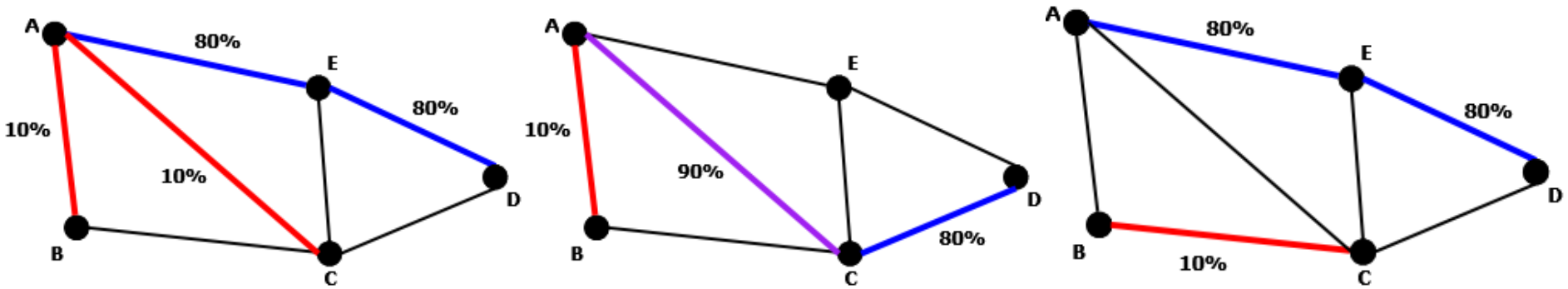

↪ Network robustness to unpredictable traffic demand growth

Parameters

- $G(N, A)$
- $c_{\{ij\}}$ capacity of edge $\{i, j\} \in A$
- K set of commodities
  - each commodity $k \in K$ is characterized by its origin node o_k, its destination node d_k and its demand $b_k > 0$
- P_k set of candidate paths for commodity $k \in K$
- $\delta^{pk}_{\{ij\}}$ binary parameter
  - indicates if edge $\{i, j\}$ is in path $p \in P_k$

Decision variables

- $\mu_{\{ij\}} \in [0,1]$ – load in edge $\{i,j\} \in A$
- $\varphi_k^p \in \{0,1\}$ – indicates if candidate $p \in P_k$ is chosen as the path for routing $k \in K$

Network Load Balancing Optimization

Mathematical formulation

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Constraints

$$\sum_{p \in P_k} \varphi_k^p = 1 \qquad \forall k \in K \tag{1}$$

$$\sum_{k \in K} \sum_{p \in P_k} b_k \delta_{\{ij\}}^{pk} \varphi_k^p = c_{\{ij\}} \mu_{\{ij\}} \qquad \forall \{i,j\} \in A \tag{2}$$

$$\mu_{\{ij\}} \in [0,1],\ \varphi_k^p \in \{0,1\} \tag{3}$$

Network Load Balancing Optimization

Mathematical formulation

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↪ Vector $\mathbf{a} = (a_1, ..., a_m)$ is lexicographically smaller than $\mathbf{b} = (b_1, ..., b_m)$ if $a_1 < b_1$ or if there exists $k \in \{1, ..., m-1\}$ such that $a_i = b_i$ for $i \leq k$ and $a_{k+1} < b_{k+1}$

$(1, 3, 2)$ is lexicographically smaller than $(1, 5, 1)$

Network Load Balancing Optimization

Solution approach

- Non-linear problem $\Rightarrow$ can be solved as a sequence of MILPs

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- Hybridization of column generation and metaheuristics

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$\hookrightarrow$ Hybrid CG and GRASP+PR

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$\hookrightarrow$ Search space constrained to paths obtained by CG

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4. $\quad [\mu] \leftarrow \text{GRASP+PR}(\Omega)$
5. **end**

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↪ If Φ does not have good quality paths $\Rightarrow$ add new paths to the search space

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$\hookrightarrow$ If the search space grows too much, the search can be jeopardized $\Rightarrow$ remove some paths

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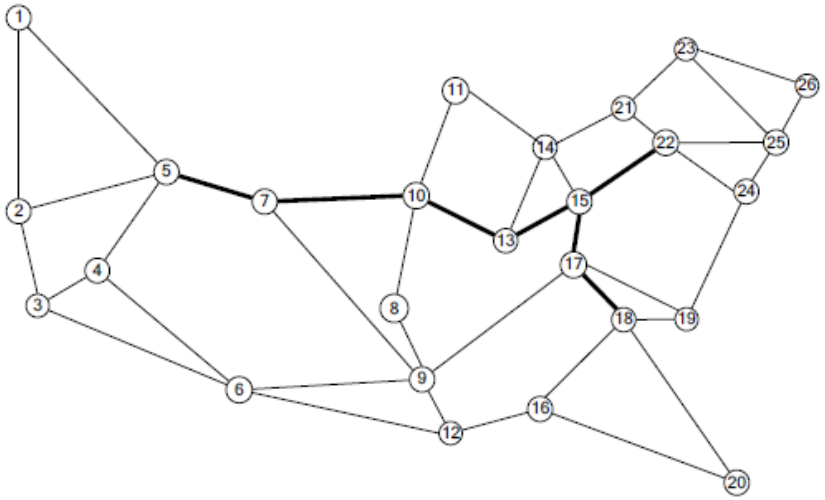
8. $\quad \Theta \leftarrow \Theta \cup \Phi$
9. $\quad \Omega \leftarrow \Omega \cup \Phi \cap E$
10. **end**

Computational Results

NSF network configurations



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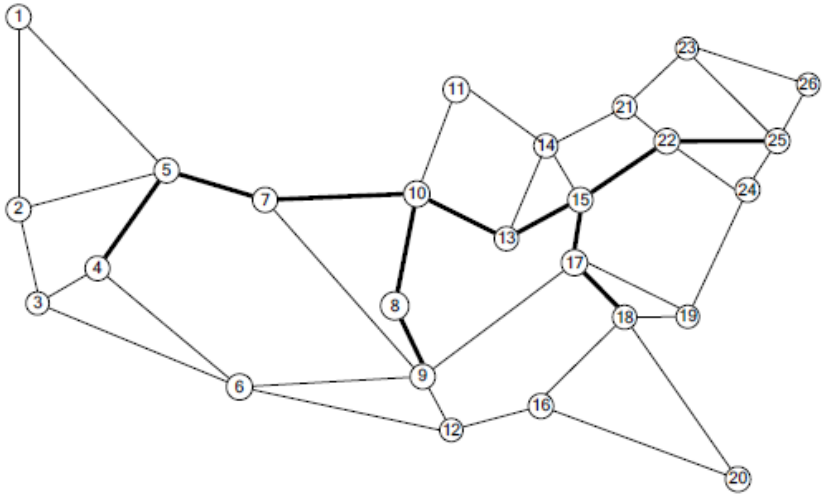

NSF''

1 Gbps; 10 Gbps in bold

Computational Results

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- Half of each group have 325 commodities (H) and half have 190 commodities (L)

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- For each instance a demand matrix was randomly generated
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- Half of each group have 325 commodities (H) and half have 190 commodities (L)
- Applying CG to the linear relaxation of the load balancing problem:
  - for each instance, the number of paths generated on average per commodity is ≈ 4

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Integer RMP

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  - 4 NSF' instances and 6 NSF" instances
  - half of these instances are of group H
  - at least 3 worst loads were solved and are optimal
  - integer RMP values coincide with LP values

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- Group B:
  - all the other cases
  - the worst load value does not coincide with the LP value
  - for 2 NSF' instances and 2 NSF" instances the worst load was not even solved within the timelimit
  - may be optimal or not

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↪ The LP paths contain near optimal solutions for group G

Computational Results

Hybrid CG and GRASP+PR

Search space is not modified (no paths are added):

- Results between PR with incumbent (elite size = 1) and PR with elite list

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- Elite list: size = 20; distance = 30%
- Timelimit = 60 seconds; 10 runs for each instance and PR case
- PR with incumbent:
  - for instances of group G the solutions are of good quality ⇒ almost always the 3 worst link loads are optimal

Computational Results

Hybrid CG and GRASP+PR

- PR with elite list:

Instance Sets	Improvement
Global	38.1%
B	56.4%
G	12.8%
H	56.4%
L	17.4%

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Instance Sets	Improvement
Global	38.1%
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↪ Overall improvement using elite list

↪ Improvement is more significant for groups B and H

↪ Average occupancy of elite list is between 4 and 5 $\Rightarrow$ PR does not penalize performance

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Hybrid CG and GRASP+PR

	P_1 without exclusion of paths		P_1 with exclusion of paths	
Sets	Modifications	Improvement	Modifications	Improvement
Global	7.2	63.2%	7.1	70.3%
B	6.3	56.4%	6.4	56.4%
G	8.6	77.8%	8.0	85.7%
H	6.7	43.9%	6.2	56.4%
L	7.8	85.7%	7.9	85.7%

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L	7.8	85.7%	7.9	85.7%

- ↪ Number of added paths does not increase significantly the size of the restricted search space
- ↪ Number of removed paths is of the same order of magnitude of the number of added paths

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- ↪ Number of added paths does not increase significantly the size of the restricted search space
- ↪ Number of removed paths is of the same order of magnitude of the number of added paths
- ↪ Adding paths using P_1 shows improvement; slightly better performance when exclusion of paths is used
- ↪ Significant improvements for groups G and L

Computational Results

Hybrid CG and GRASP+PR

P_2:

- ↪ Number of modifications of P_2 similar to P_1
- ↪ P_2 adds much less paths and excludes much more paths $\Rightarrow$ the restricted search space decreases in size
- ↪ P_2 shows slight improvement when exclusion of paths is used
- ↪ P_2 performs worse than P_1

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P_1/P_2 combined version

Computational Results

Hybrid CG and GRASP+PR

	P_1/P_2 without exclusion of paths		P_1/P_2 with exclusion of paths	
Sets	Modifications	Improvement	Modifications	Improvement
Global	4.9	56.4%	5.0	94.1%
B	4.4	56.4%	4.4	112,5%
G	5.6	50.0%	5.8	70.3%
H	4.8	85.7%	4.8	112.5%
L	5.2	32.6%	5.1	77.8%

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Sets	Modifications	Improvement	Modifications	Improvement
Global	4.9	56.4%	5.0	94.1%
B	4.4	56.4%	4.4	112,5%
G	5.6	50.0%	5.8	70.3%
H	4.8	85.7%	4.8	112.5%
L	5.2	32.6%	5.1	77.8%

- ↪ Significant decrease in the number of modifications of the search space
- ↪ Number of added paths similar to P_1
- ↪ Number of excluded paths significantly lower than P_1
- ↪ Combined performance with path exclusion is the best

Conclusions

Hybrid CG and GRASP+PR applied to network load balancing

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  - favors especially instances from groups B and H
- To improve furthermore: add new paths using P_1 and/or P_2
  - P_1 results in overall better solutions especially when some paths are removed
    - overall improvement of 70.3%
    - favors especially instances of groups G and L

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- PR shows improvement when using elite list with a distance large enough $\Rightarrow$ average occupancy of list should be low
  - favors especially instances from groups B and H
- To improve furthermore: add new paths using P_1 and/or P_2
  - P_1 results in overall better solutions especially when some paths are removed
    - overall improvement of 70.3%
    - favors especially instances of groups G and L
  - P_2 has poor overall performance

Conclusions

Hybrid CG and GRASP+PR applied to network load balancing

- GRASP+PR performed on restricted search space $\Rightarrow$ search is more efficient
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- To improve furthermore: add new paths using P_1 and/or P_2
  - P_1 results in overall better solutions especially when some paths are removed
    - overall improvement of 70.3%
    - favors especially instances of groups G and L
  - P_2 has poor overall performance
  - P_1/P_2 combined version has the best overall performance especially when some paths are removed
    - overall improvement of 94.1%
    - favors significantly all groups especially B and H

Conclusions

Hybrid CG and GRASP+PR: future work

- Apply heuristic proposal to more complex load balancing problems:
  - Load balancing using multiple spanning trees
    - ↪ D. Santos, A. de Sousa, F. Alvelos, M. Pióro, Optimization of Network Load Balancing in Multiple Spanning Tree Routing Networks, Telecommunication Systems Journal, 48(1-2), pp 109-124, Springer, 2011
  - Load balancing with path protection
    - ↪ A. de Sousa, D. Santos, P. Matos and J. Madeira, Load balancing optimization of capacitated networks with path protection, Electronic Notes, 36, pp. 1249-1256, 2010

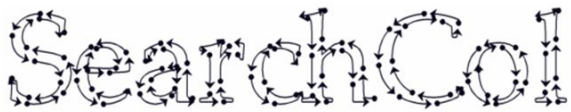
Conclusions

Hybrid CG and GRASP+PR: SearchCol

- Complementary work to

↪ D. Santos, A. de Sousa, F. Alvelos, M. Pióro, Link Load Balancing Optimization of Telecommunication Networks: a Column Generation based Heuristic Approach, NETWORKS 2010, Warsaw, Poland, September, 2010

↪ F. Alvelos, A. de Sousa, D. Santos, SearchCol: Metaheuristic Search by Column Generation, 7th International Workshop on Hybrid Metaheuristics, Vienna, Austria, October, 2010



Metaheuristic Search by Column Generation

http://searchcol.dps.uminho.pt